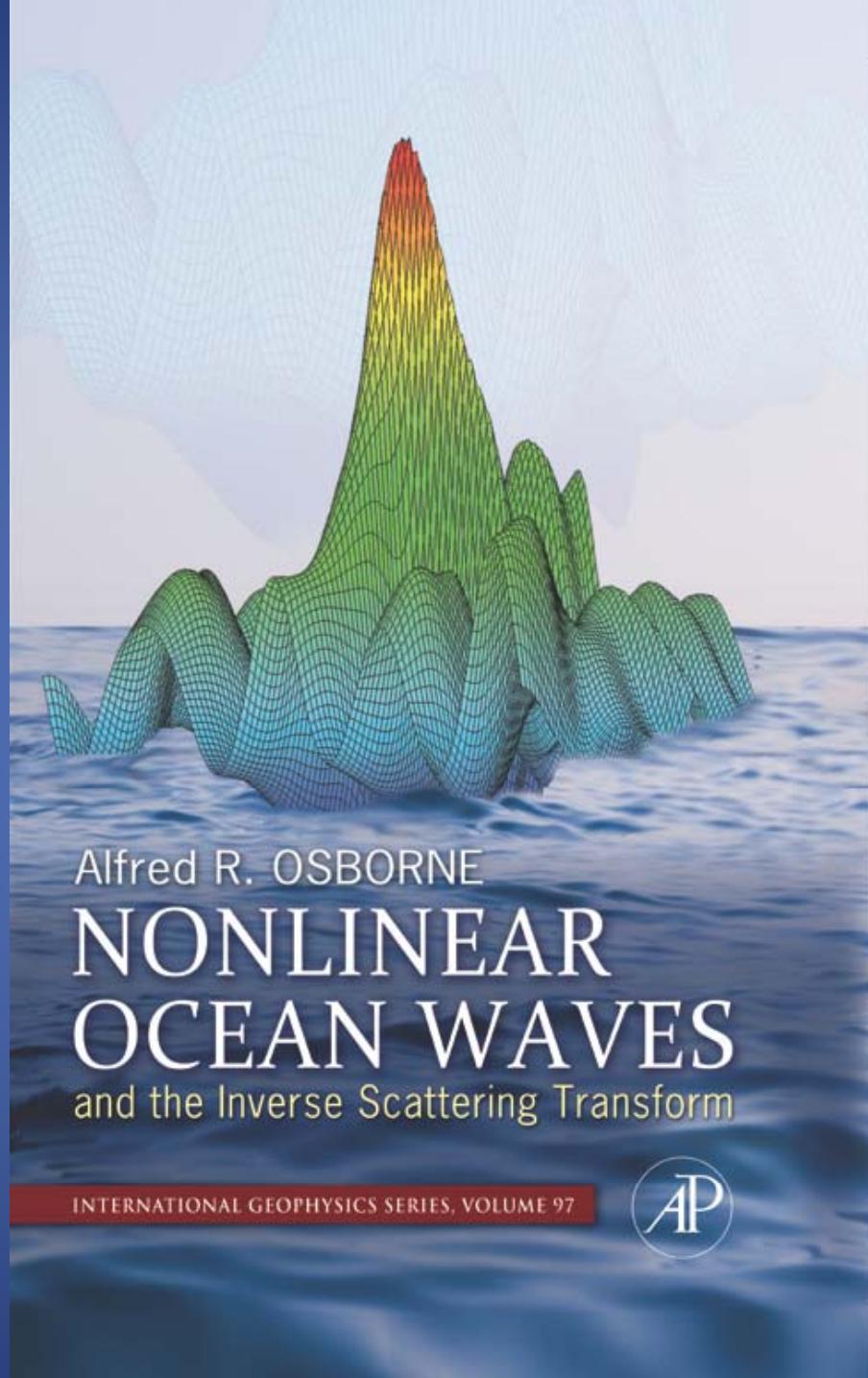


Uncertainty in Internal Wave Fields: Coherent Structures and Nonlinear Waves

A. R. Osborne

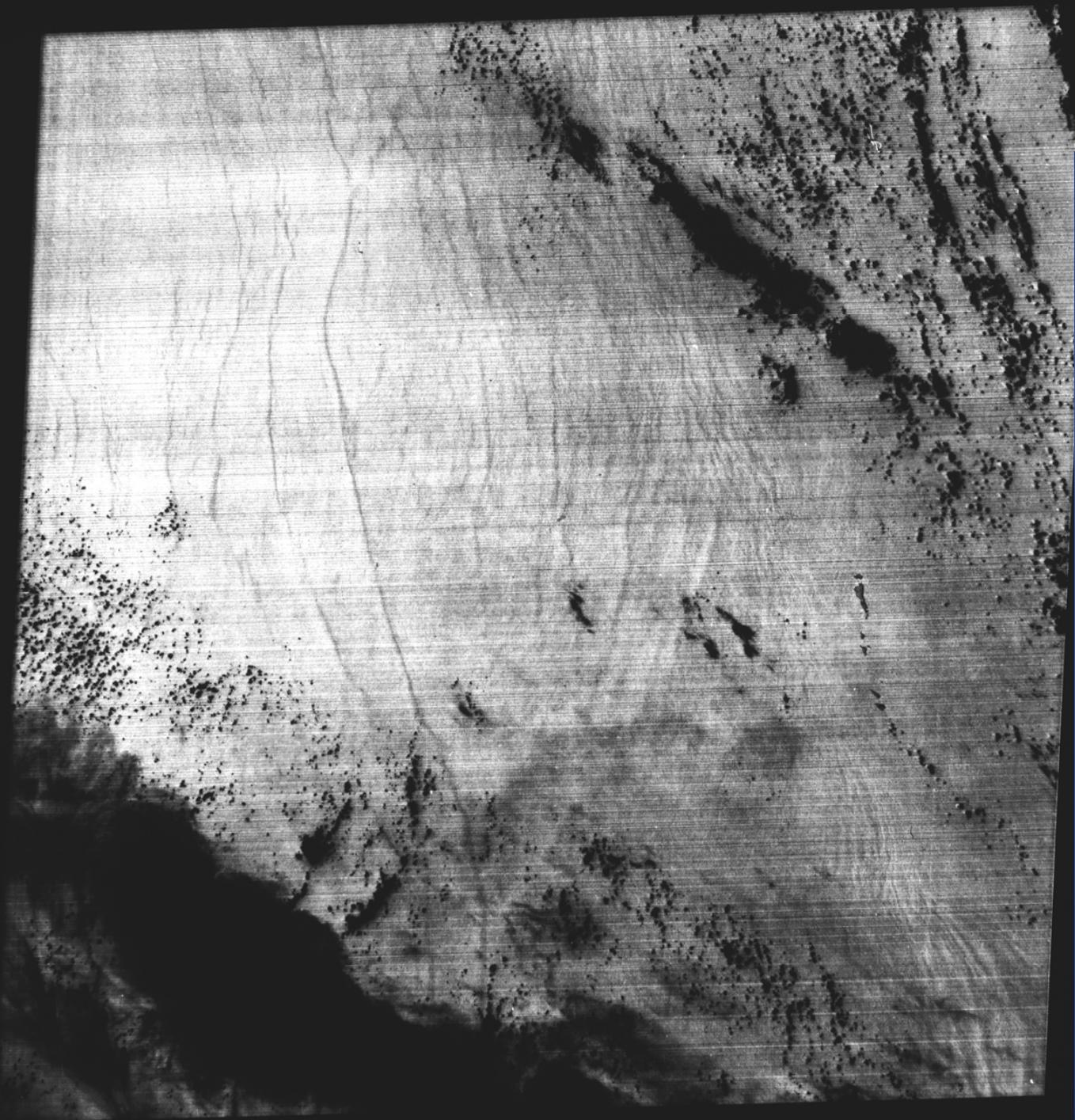
Nonlinear Waves Research Corporation, Arlington, VA
Department of Physics, Torino, Italy

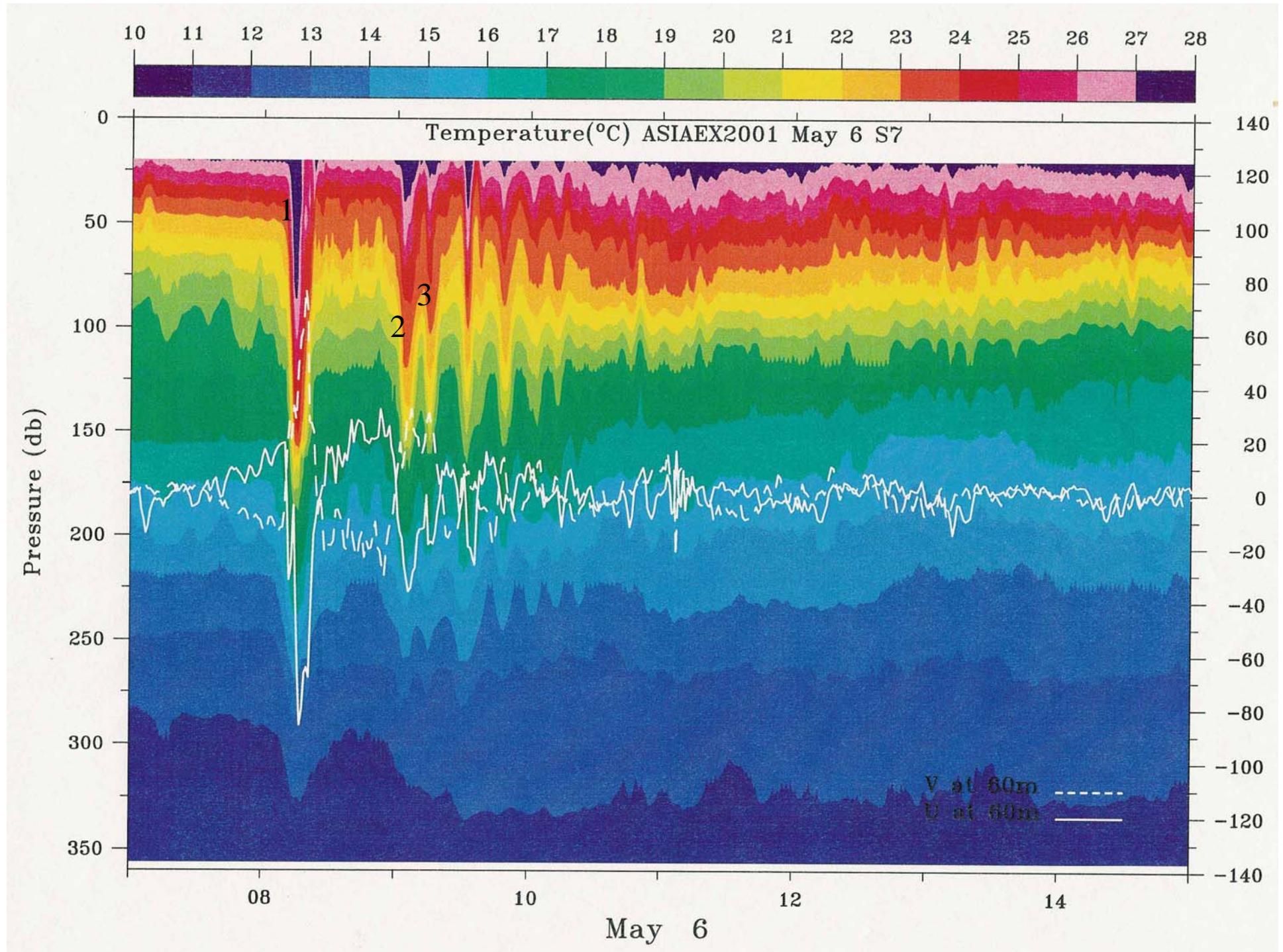


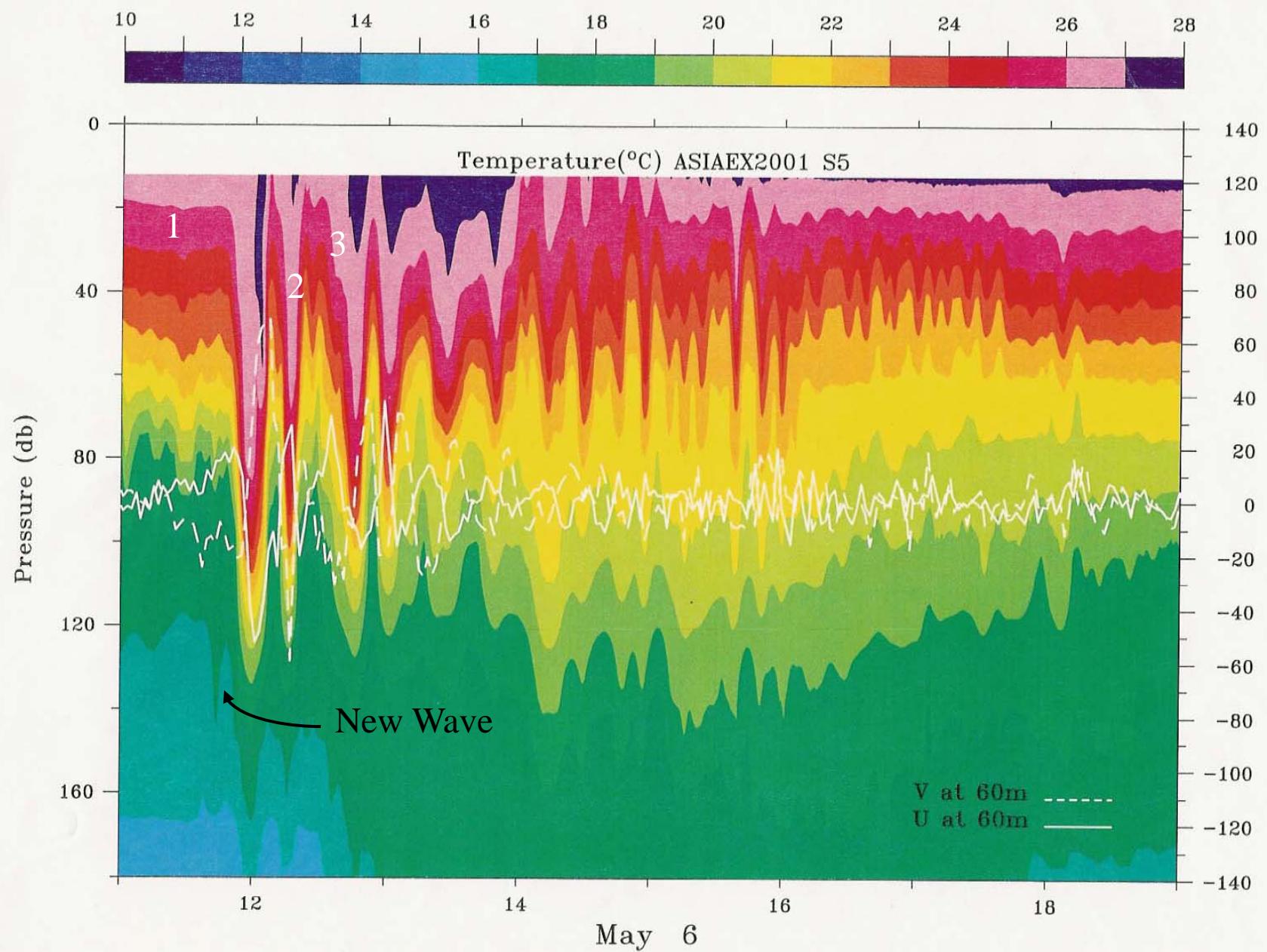


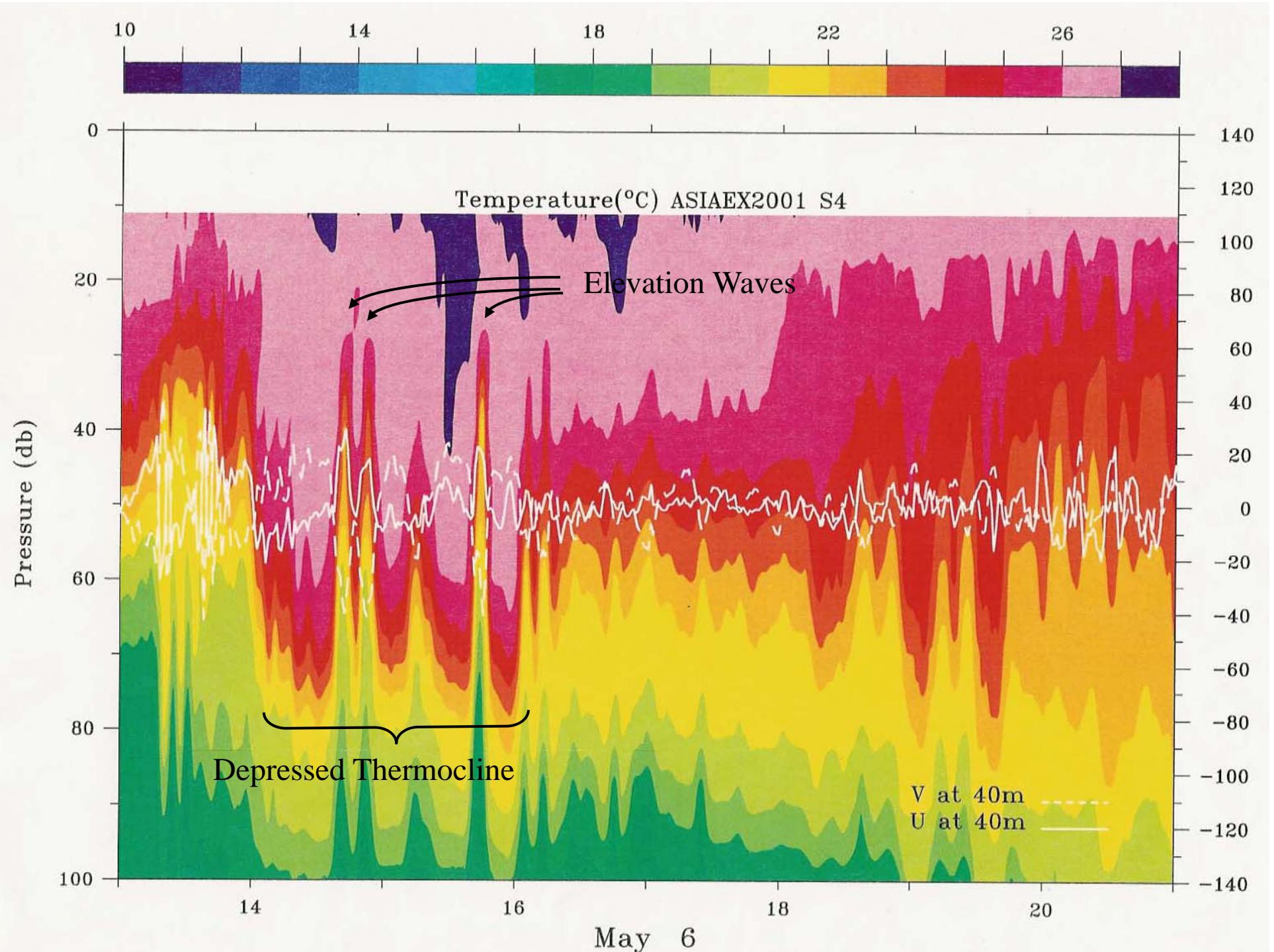


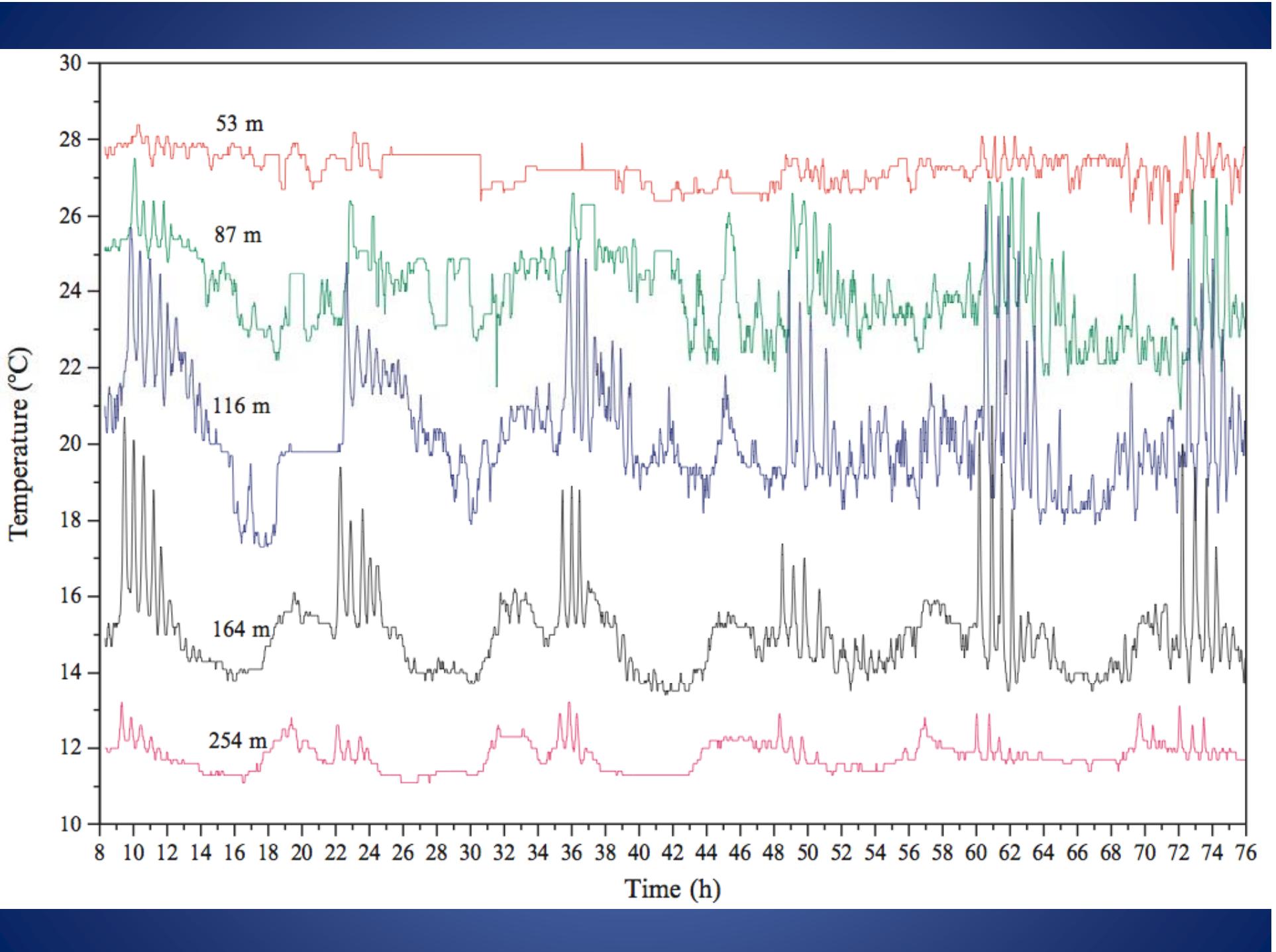












Equations in One Space and One Time Dimensions

KdV Equation

$$\eta_t + c_o \eta_x + \alpha \eta \eta_x + \beta \eta_{xxx} = 0$$

Gardner Equation

$$\eta_t + c_o \eta_x + \alpha \eta \eta_x + \beta \eta_{xxx} = \delta \eta^2 \eta_x$$

Equations in Two Space and One Time Dimensions

KP Equation

$$\eta_t + c_o \eta_x + \alpha \eta \eta_x + \beta \eta_{xxx} + \gamma \partial^{-1} \eta_{yy} = 0$$

KP Gardner Equation

$$\eta_t + c_o \eta_x + \alpha \eta \eta_x + \beta \eta_{xxx} + \gamma \partial^{-1} \eta_{yy} = \delta \eta^2 \eta_x$$

2+1 Gardner Equation

$$\begin{aligned}\eta_t + c_o \eta_x + \alpha \eta \eta_x + \beta \eta_{xxx} + \gamma \partial^{-1} \eta_{yy} = \\ \delta \eta^2 \eta_x + \rho \eta_x \partial_x^{-1} \eta_y\end{aligned}$$

$$c_0 = \sqrt{gh}, \quad \alpha = \frac{3c_0}{2h}, \quad \beta = \frac{c_0 h^2}{6}, \quad \gamma = \frac{c_0}{2}, \quad \delta = \frac{15c_0}{8h^2}, \quad \rho = \sqrt{\frac{15}{8}} \frac{c_0}{h}$$

The Hierarchy Leads to Euler!

1+1 Dimensions

KdV Equation

Gardner Equation

2+1 Dimensions

KP Equation

KP Gardner Equation

2+1 Gardner Equation

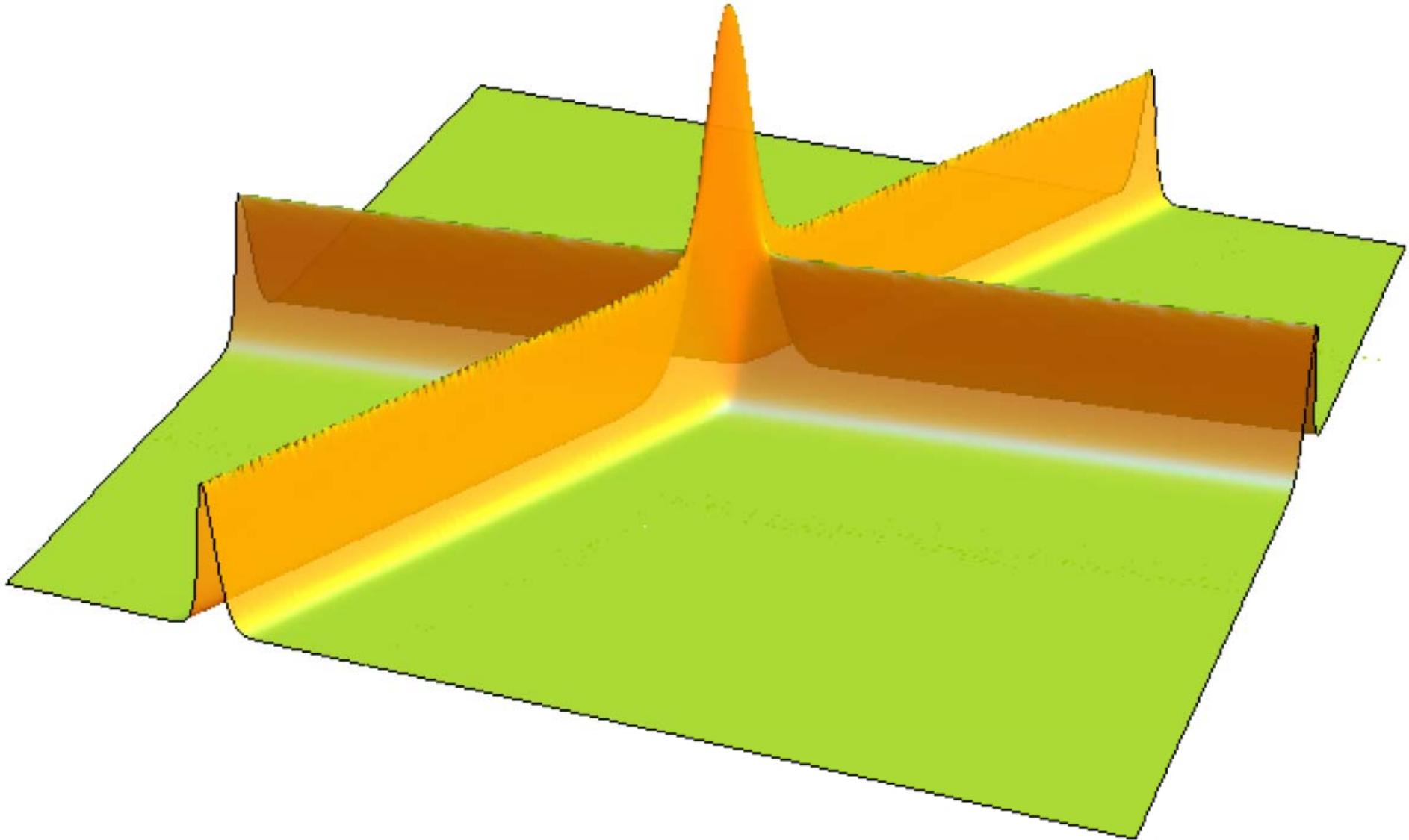
Euler Equations

Kadomsev-Petviashvili Equation: KdV Equation Plus Directionality

$$\eta_t + c_o \eta_x + \alpha \eta \eta_x + \beta \eta_{xxx} + \gamma \partial^{-1} \eta_{yy} = 0$$

$$c_0 = \sqrt{gh}, \quad \alpha = \frac{3c_0}{2h}, \quad \beta = \frac{c_0 h^2}{6}, \quad \gamma = \frac{c_0}{2}$$

KP Equation: Two Cnoidal Waves Plus Mach Stem



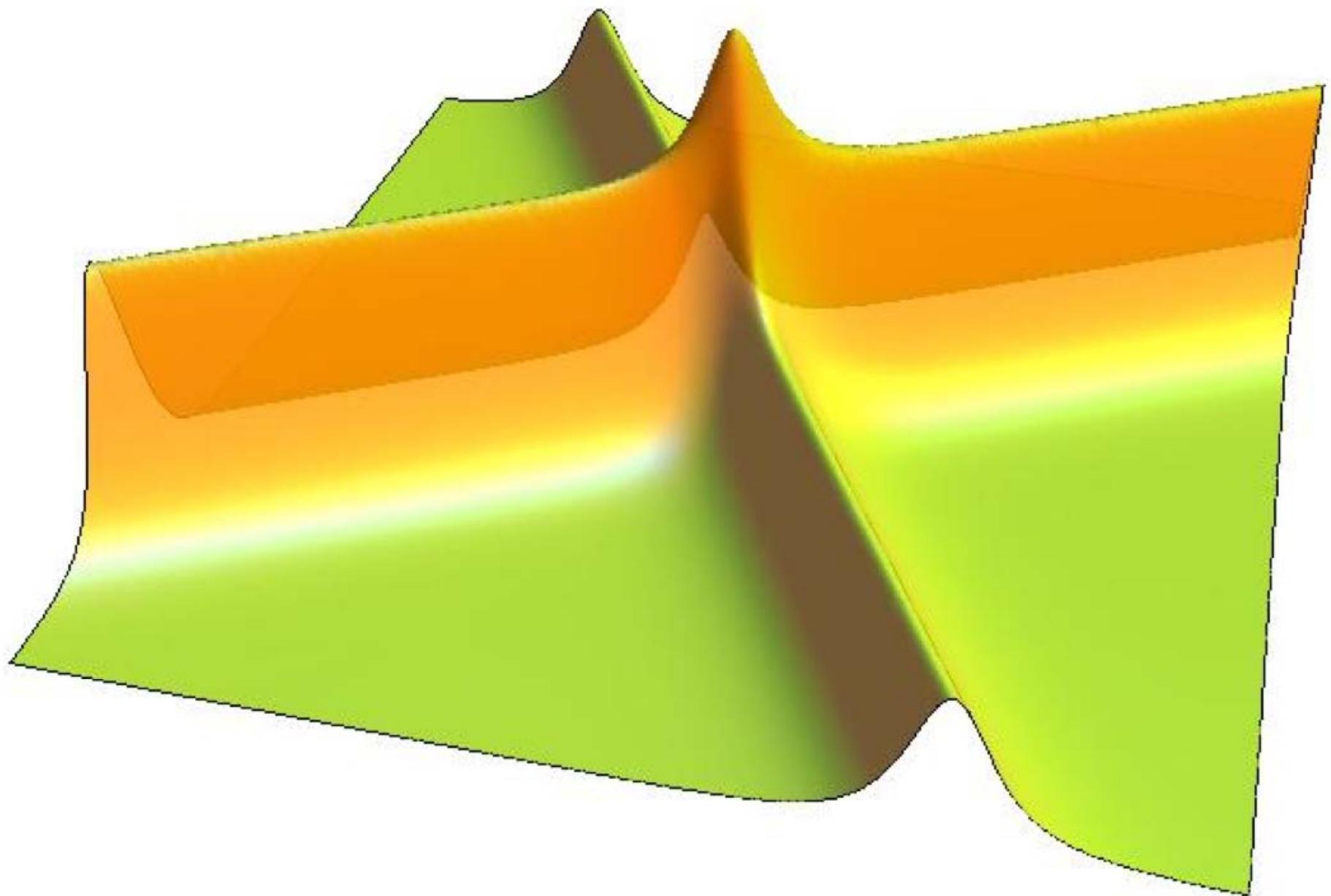


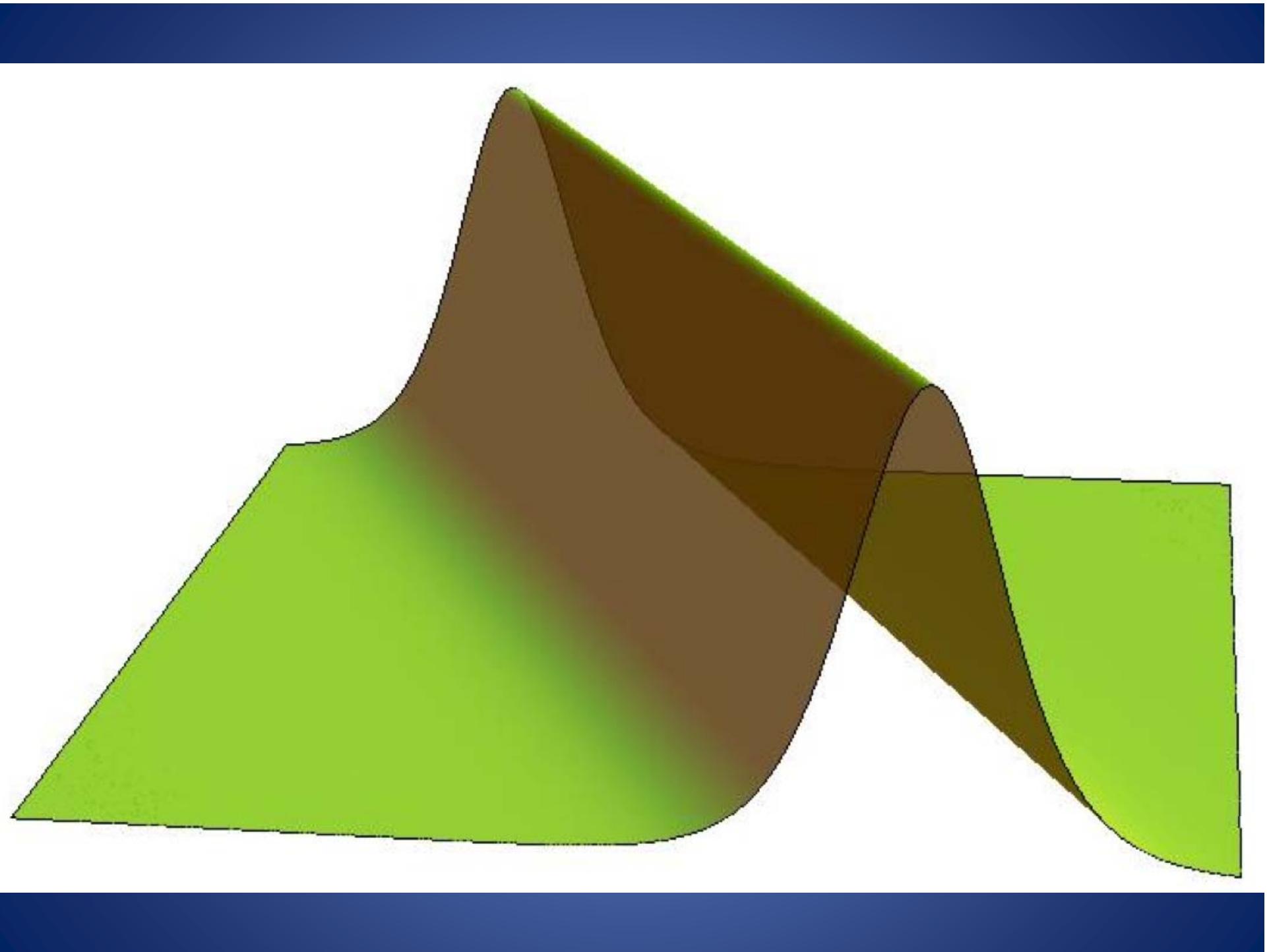
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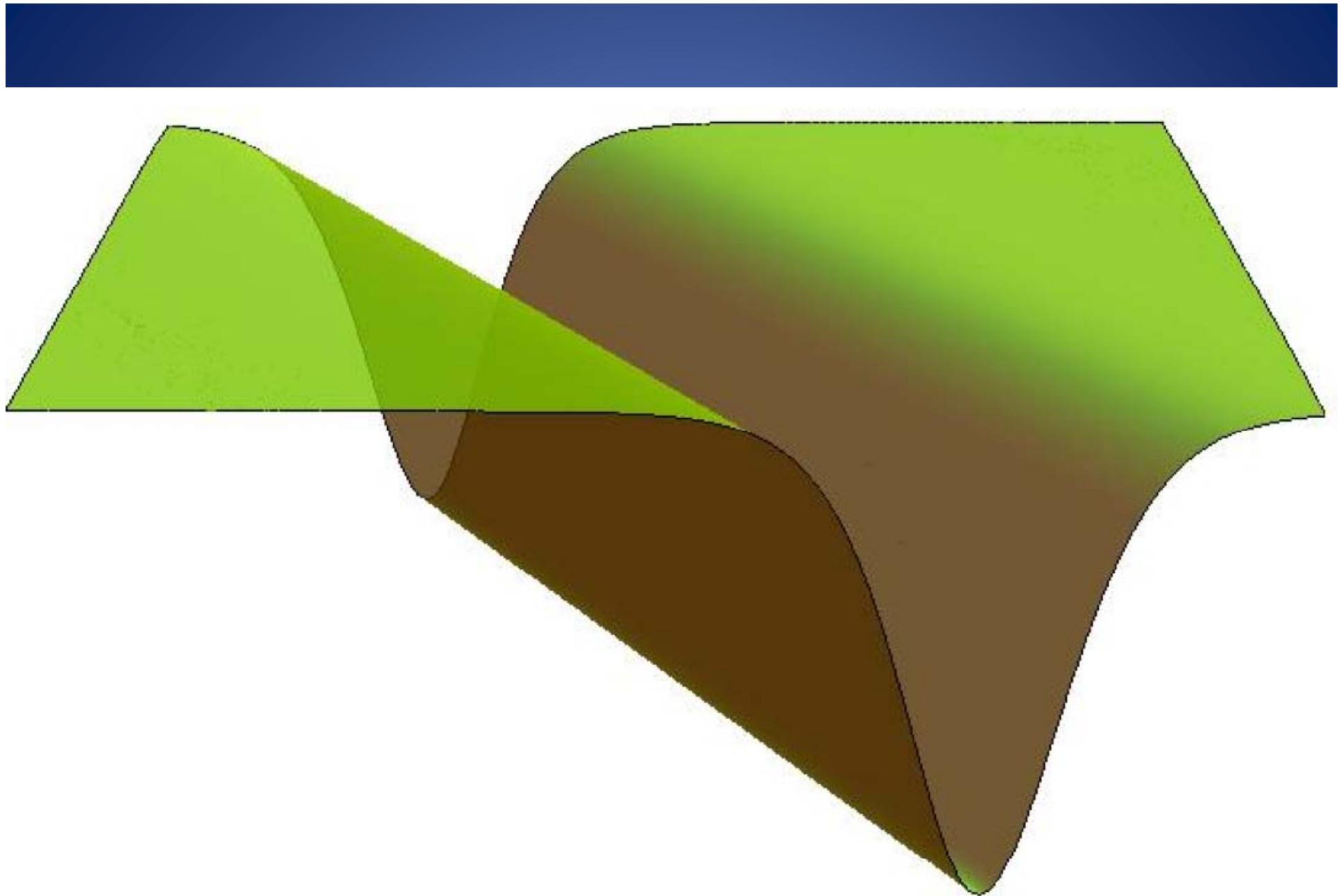
Gardner Equation in 2 Space and 1 Time Dimensions

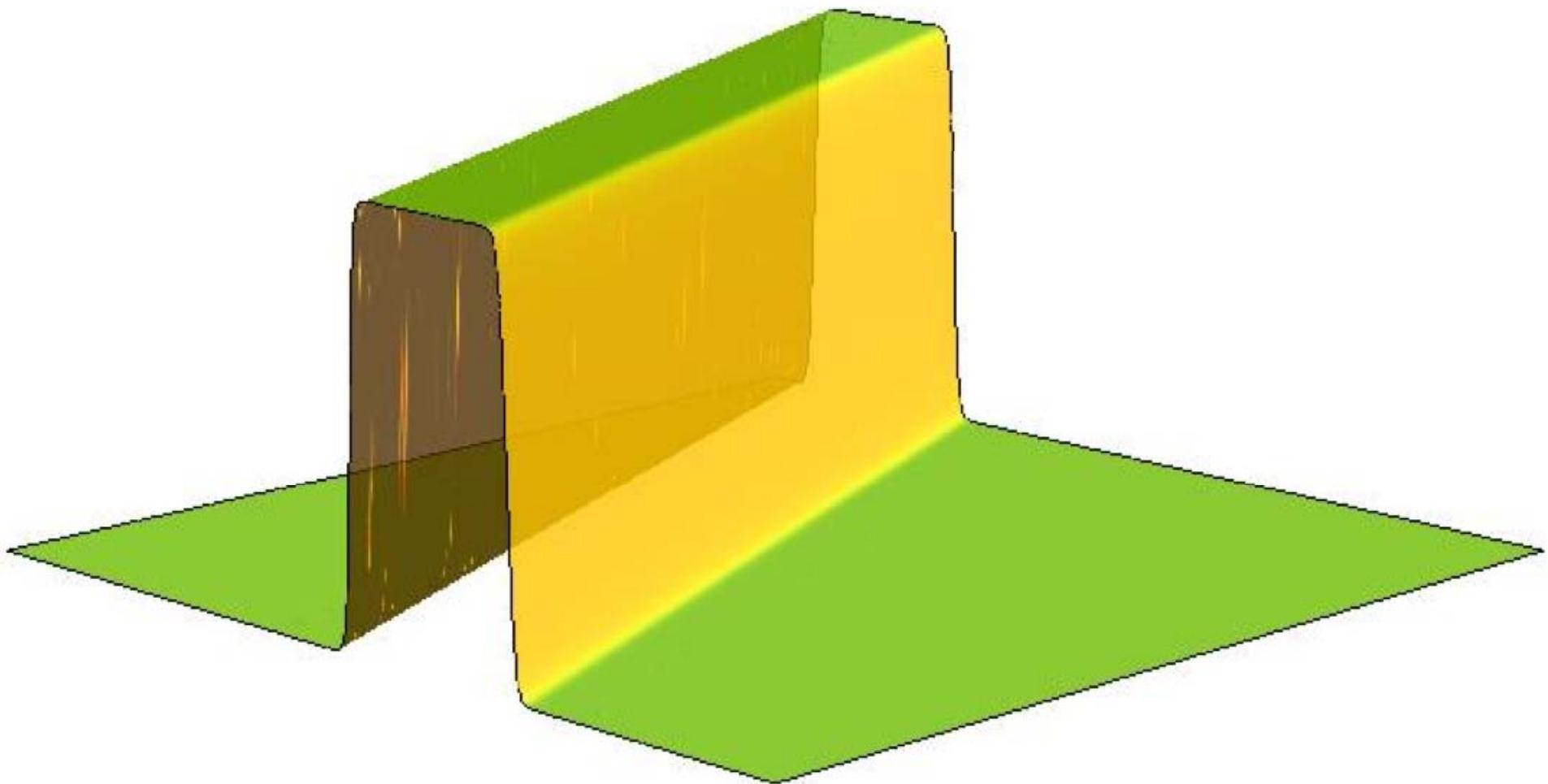
$$\eta_t + c_0 \eta_x + \alpha \eta \eta_x + \beta \eta_{xxx} + \gamma \partial_x^{-1} \eta_{yy} = \delta \eta^2 \eta_x + \rho \eta_x \partial_x^{-1} \eta_y$$

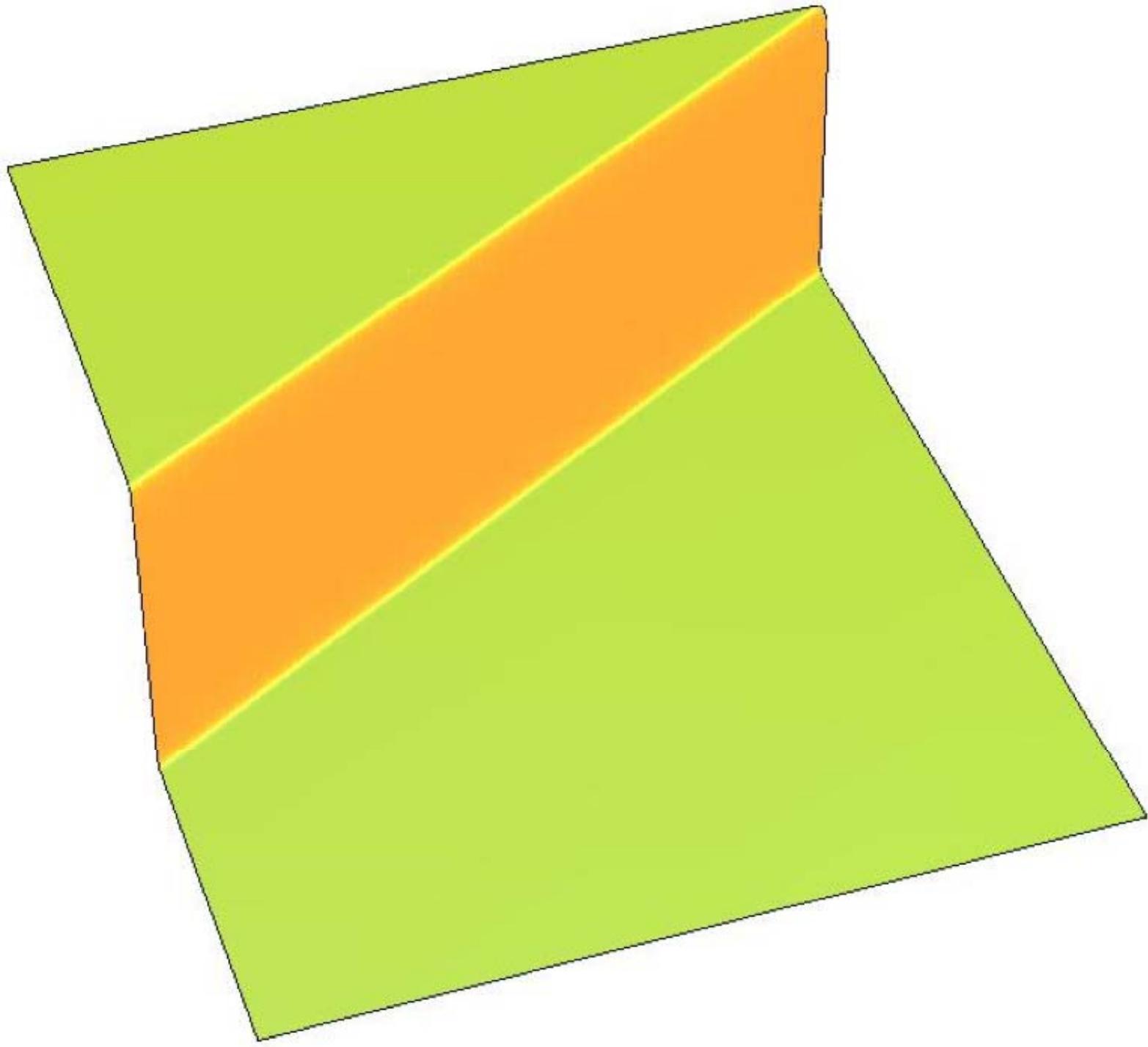
$$c_0 = \sqrt{gh}, \quad \alpha = \frac{3c_0}{2h}, \quad \beta = \frac{c_0 h^2}{6}, \quad \gamma = \frac{c_0}{2}, \quad \delta = \frac{15c_0}{8h^2}, \quad \rho = \sqrt{\frac{15}{8}} \frac{c_0}{h}$$

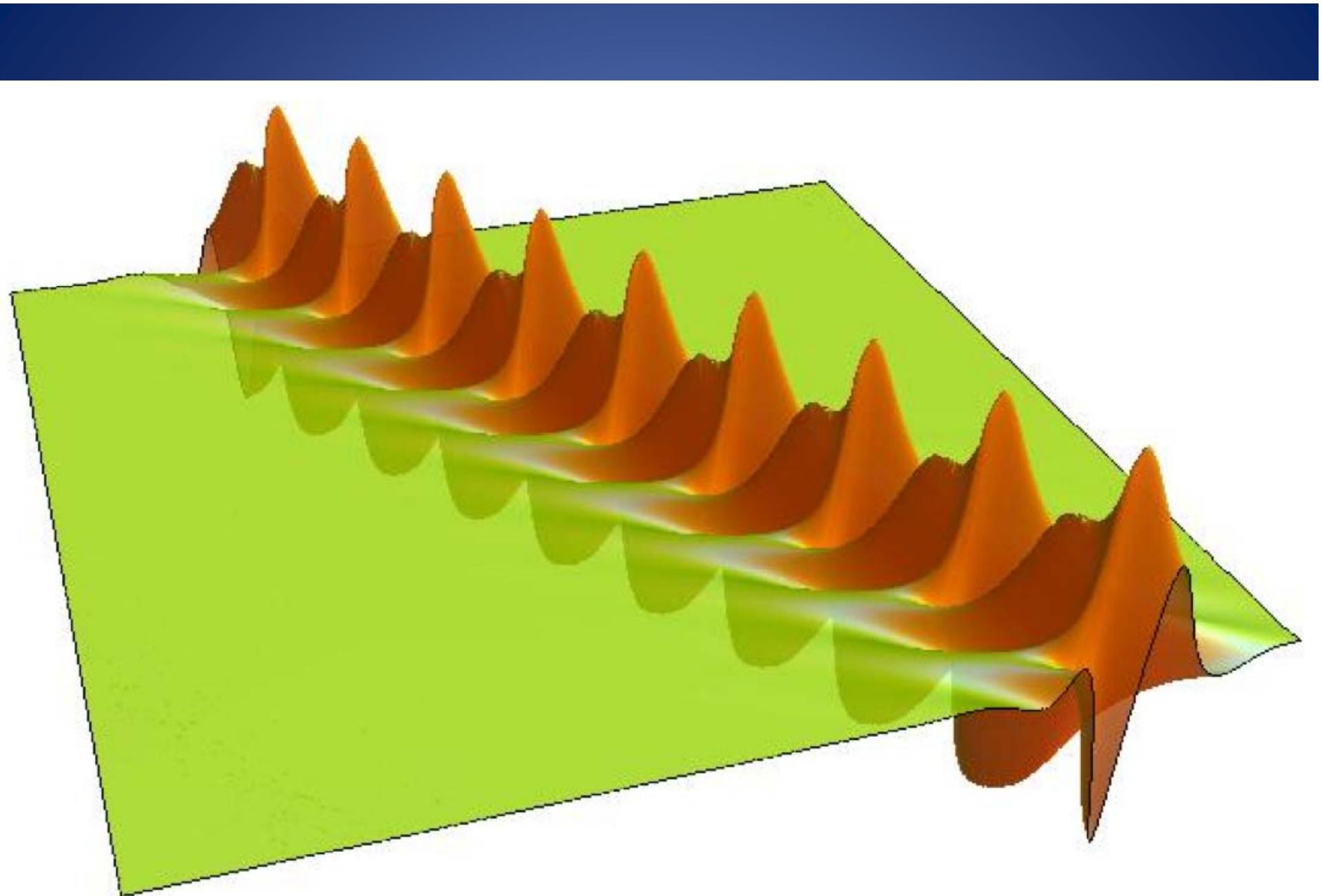


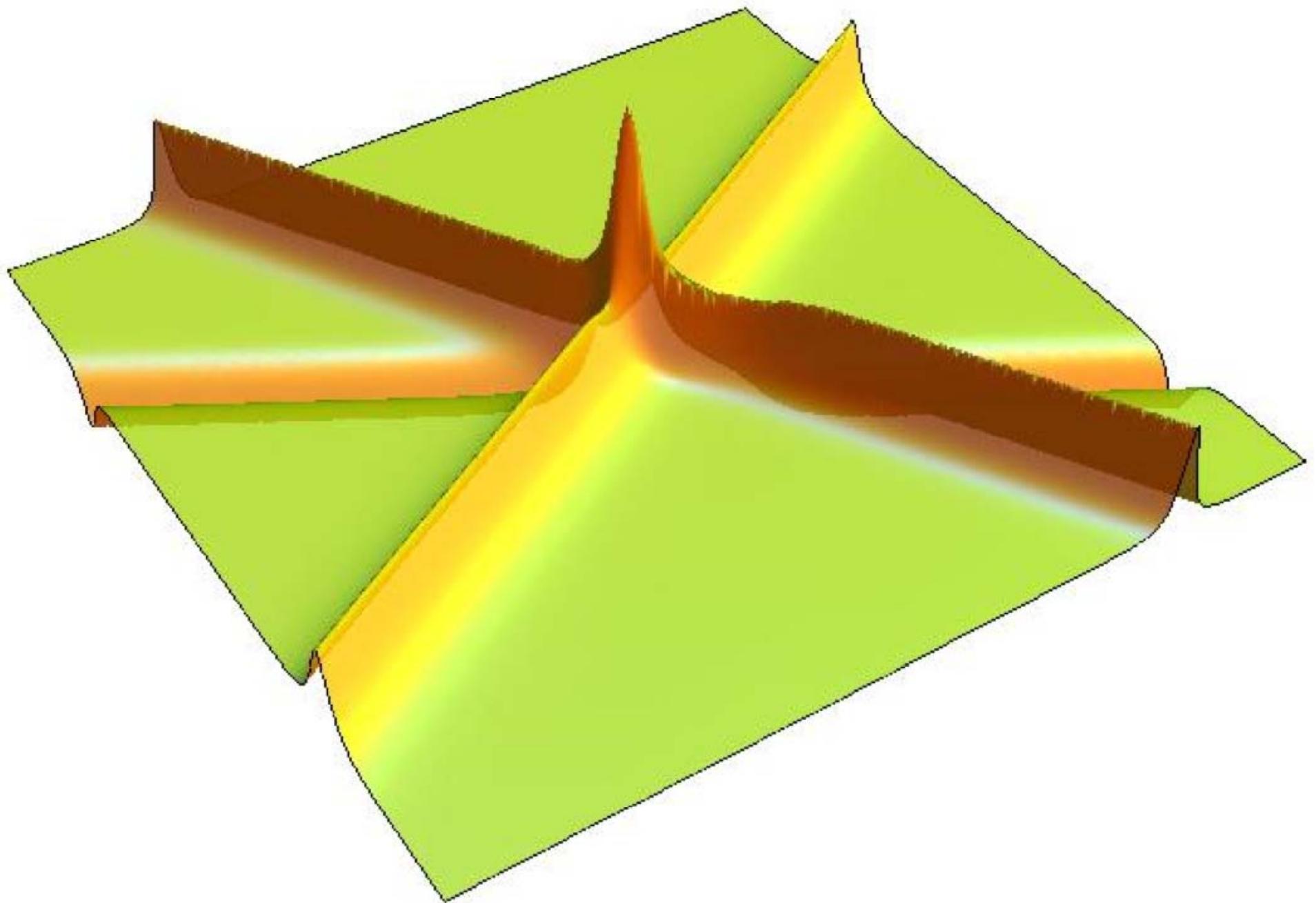


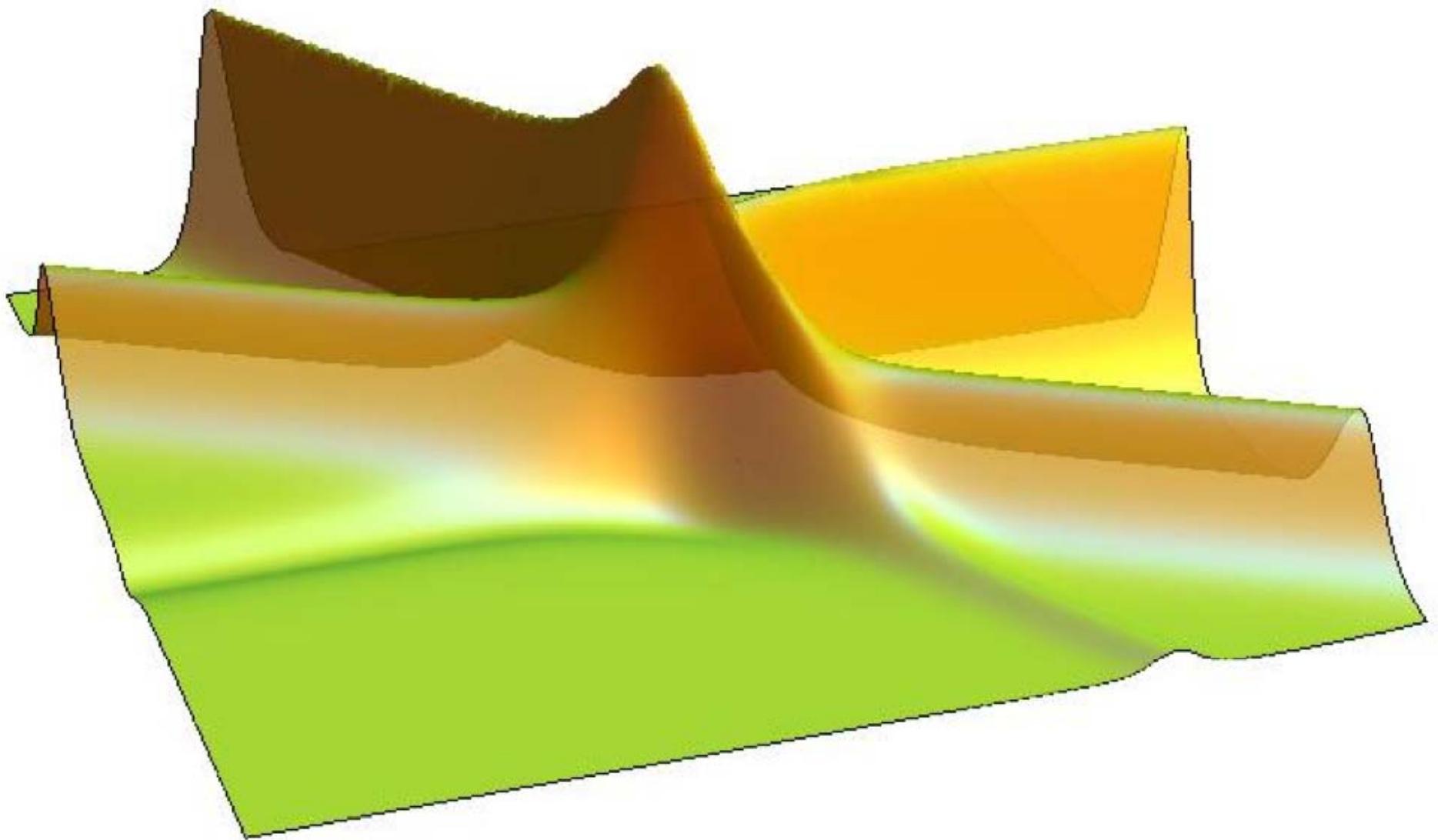














$$\nabla^2 \psi - \frac{1}{c^2(r,z)} \frac{\partial^2 \psi}{\partial t^2} = 0$$

The wave equation

$t \rightarrow -t$ Leaves wave equation unchanged

The sound source:
UUV or other moving
vehicle.

Variable sound speed $c(r,z)$

Trajectory

Registered sound time series

Transducer array

