Uncertainty in internal solitary wave modeling due to parameter variability.

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Outline

- ***1D version of the viscous shallow water equations***
- ***Relationship between the first and second EOF coefficient in presence of internal solitary waves.***
- ***Effects of variability of tidal magnitude and stratification.***
- ***Exploitation of confinement in parameter space.***
- ***Feedback loop between acoustic and internal solitary waves.***

Saint-Venant Equations
Traveling wave solution

$1D$ *viscous shallow water equations* (*aka Saint–Venant Eqs.*) *are*

$$h_\tau + (hv)_X = 0, \quad (hv)_\tau + (hv^2)_X + \frac{1}{2}g(h^2)_X = \kappa(hv_X)_X \quad (1)$$

Assuming the ansatzs $v = c_0 V(\xi)$, $h = h_e H(\xi)$, *where* $\xi := x - at$

is the wave variable resduces Sys. (1), *on integrating Eq.* (1), *wrt* ξ,

to $V = c(1 - H^{-1})$ *and* $\varepsilon H V' = H^2 - 2cHV + 2HV^2 - 1, \quad (2)$

where $H > 0$, $c = a/c_0$, *a prime denotes* $d/d\xi$, *and* $\varepsilon \propto \kappa$.

Eliminating V *from Sys.* (2) *yields the Abel equation*

$$\varepsilon c H' = H^3 - (2c^2 + 1)H + 2c^2, \quad (3)$$

which admits the equlibrium solutions: $\overline{H} = \{H_1, H_2, H_3\}$,

where $H_1 = 1$ *and* $H_{2,3} = -\frac{1}{2}[1 \mp \sqrt{1 + 8c^2}]$. $\quad (4)$

Only $H_{1,2}$ *are relevant since* $H_3 < 0$.

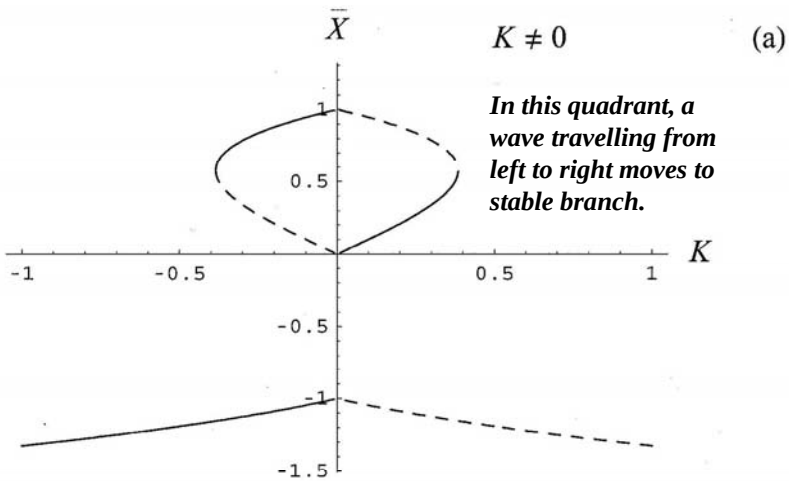
With a wavefront condition $H(0) = H_0$, *Eq.*(3) *integrates to*

$$\xi = \frac{\varepsilon c}{4(1 - c^2)}\{(\frac{6}{\sqrt{1 + 8c^2}})Arc\tanh(\frac{1 + 2H_x}{\sqrt{1 + 8c^2}})$$

$$+ \ln[\frac{(1 - H_x)^2}{H_x^2 + H_x - 2c^2}]\}$$ *evaluated as* $[H_x = H$ *minus* $H_x = H_0]$ $\quad -\infty < \xi < \infty$.

Hysteretic-type bifurcation

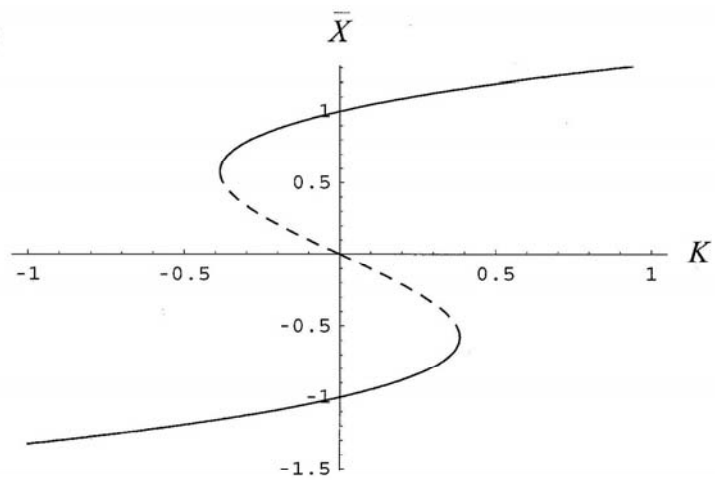
Saint-Venant equations



$X = \dfrac{H}{\sqrt{1+2c^2}}$, $K = \dfrac{2c^2\,\mathrm{sgn}(c)}{(1+2c^2)^{3/2}}$, $\zeta = (\dfrac{1+2c^2}{\varepsilon\,|c\,|})\xi$ *and* $\bar{X}$ *denotes the equlibrium values.*

Solid and broken branches correspond to stable and unstable equilibria. Only the branches in the upper half-plane are physically relevant. For waves propagating from left to right the right upper quadrant specifies the stability conditions.

Classical hysteresis example

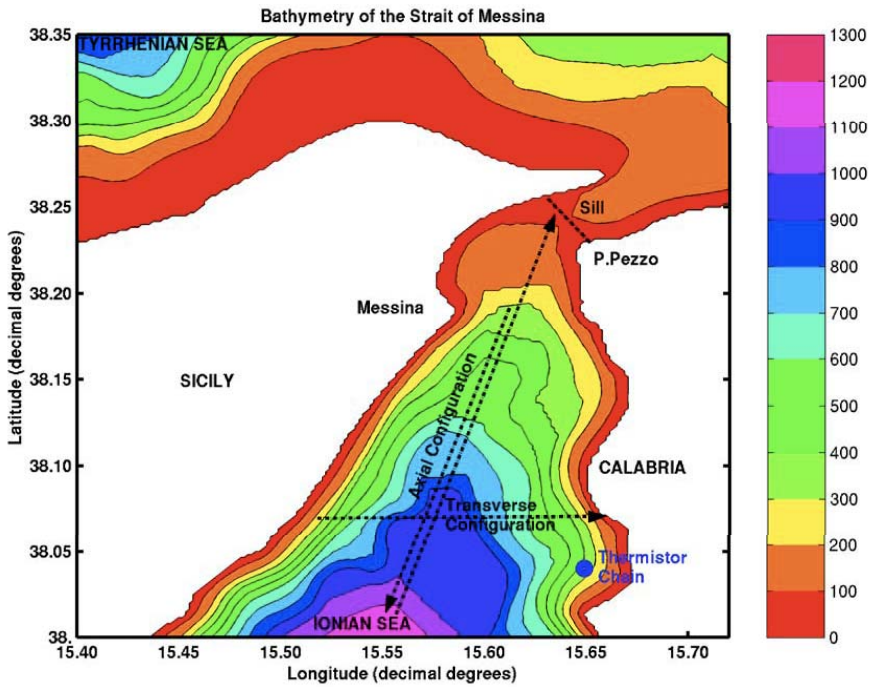


Examples of classical hysteresis are :
- ***chemical reactions (Schlogl, 1972).***
- ***biology***
- ***magnetic materials***

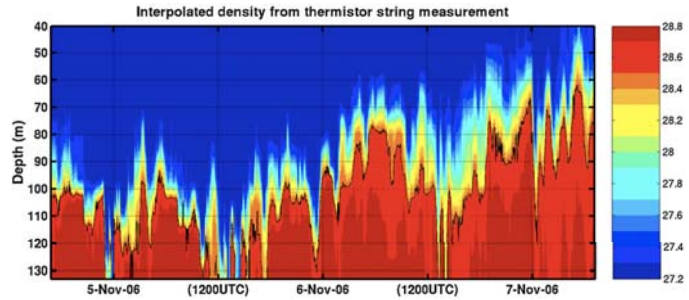
- ***We do not have the classical hysteresis.***
- ***Saint-Venant equations have bifurcation values of K as the classical case, but no hysteresis.***

Characterizing internal solitary waves with EOF analysis.

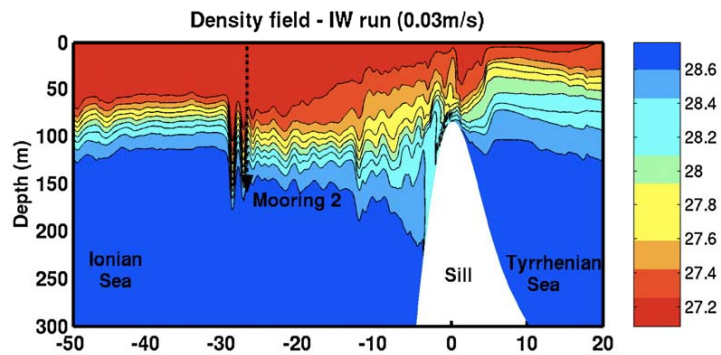
Strait of Messina & COACH06 survey



Thermistor chain data



NH model prediction

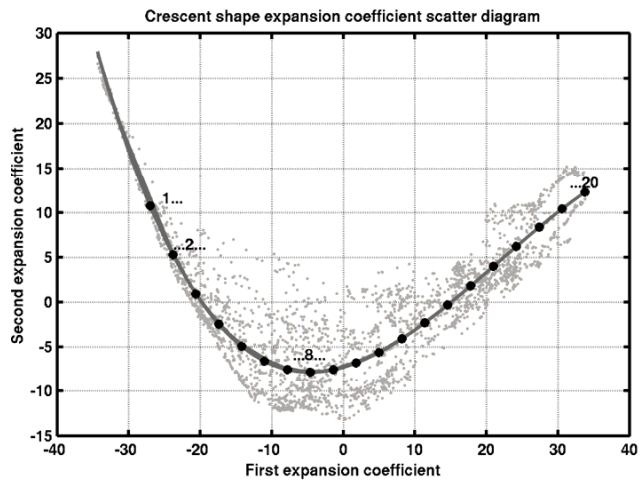


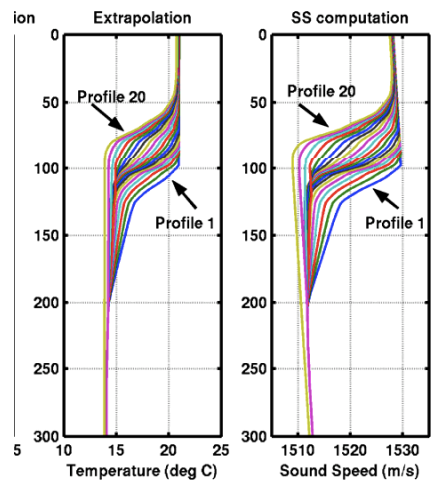
EOF decomposition

$$T(z,t) = \overline{T}(z) + \sum_{i=1}^{i=N} \alpha_i(t)\, \phi_i(z)$$

where $\overline{T}(z)$ *is the time averaged in vertical,* N *is number of modes,* $\alpha_i(t)$ *is amplitude of mode, and* $\phi_i(z)$ *is eof.*

Hysteresis relationship between 1st and 2nd eof coefficient





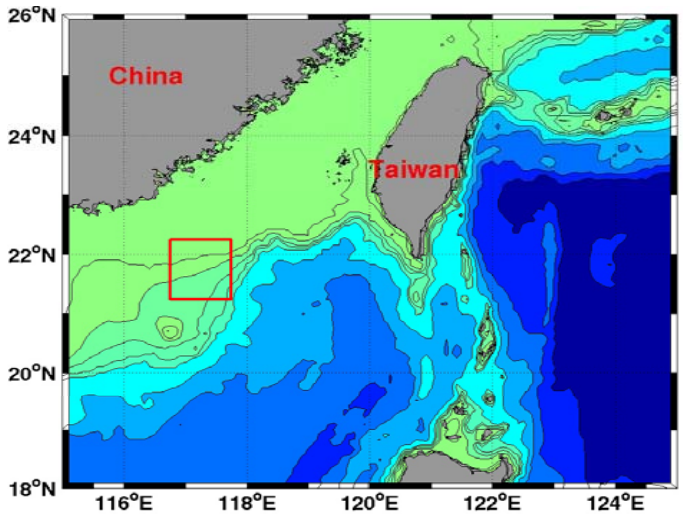
Black curve is 4^{ord} *polynomal fit.*

$$T(z,t) = \overline{T}(z) + \alpha_1(t)\,\phi_1(z) + \alpha_2(t)\,\phi_2(z) \text{ or}$$

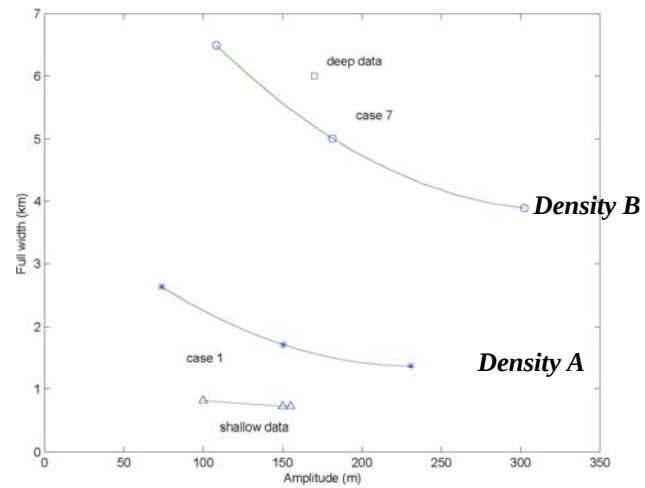
$$T(z,t) = \overline{T}(z) + \alpha_1(t)\,\phi_1(z) + f(\alpha_1)\,\phi_2(z)$$

Such a curve can be constructed for any region where internal solitary waves (ISW) are present. Using this curve ISW trains and sound speeds can be constructed and used for input into acoustical models. In model predicted ISW, uncertainties in tidal magnitude and stratification will introduce uncertainties in hysteresis relationship.

Uncertainties in predictions of solitary waves dynamics in Luzon Strait and South China Sea with Lamb 2.5 D NH model.

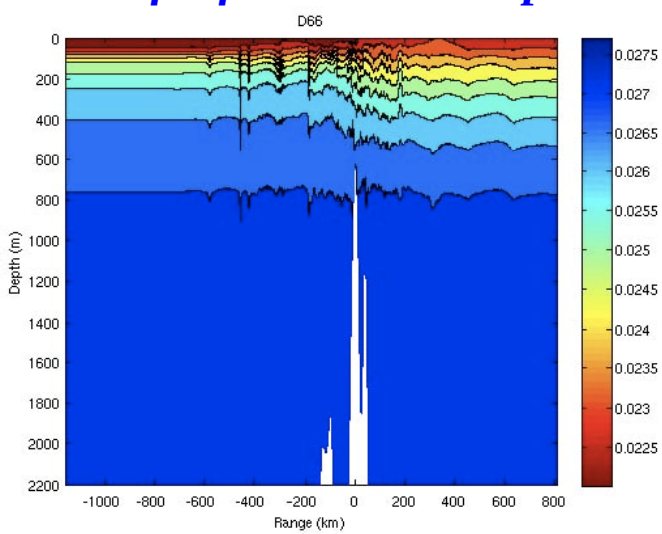


Effects of density variation.

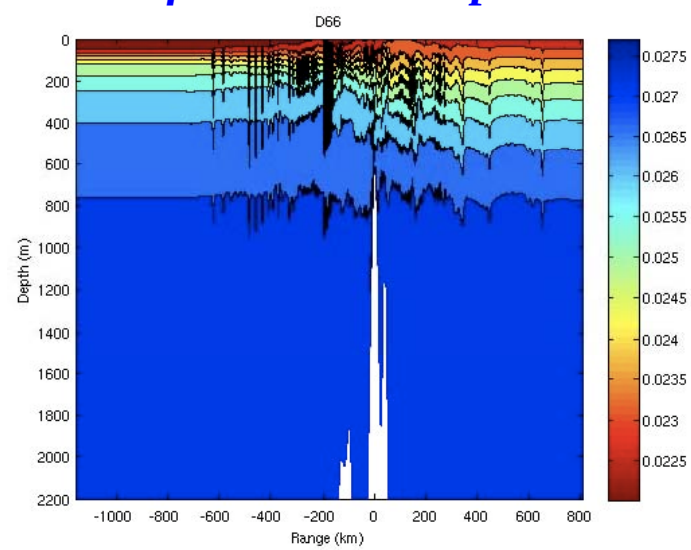


Effects of barotropic tide magnitude.

Half reference barotropic tide

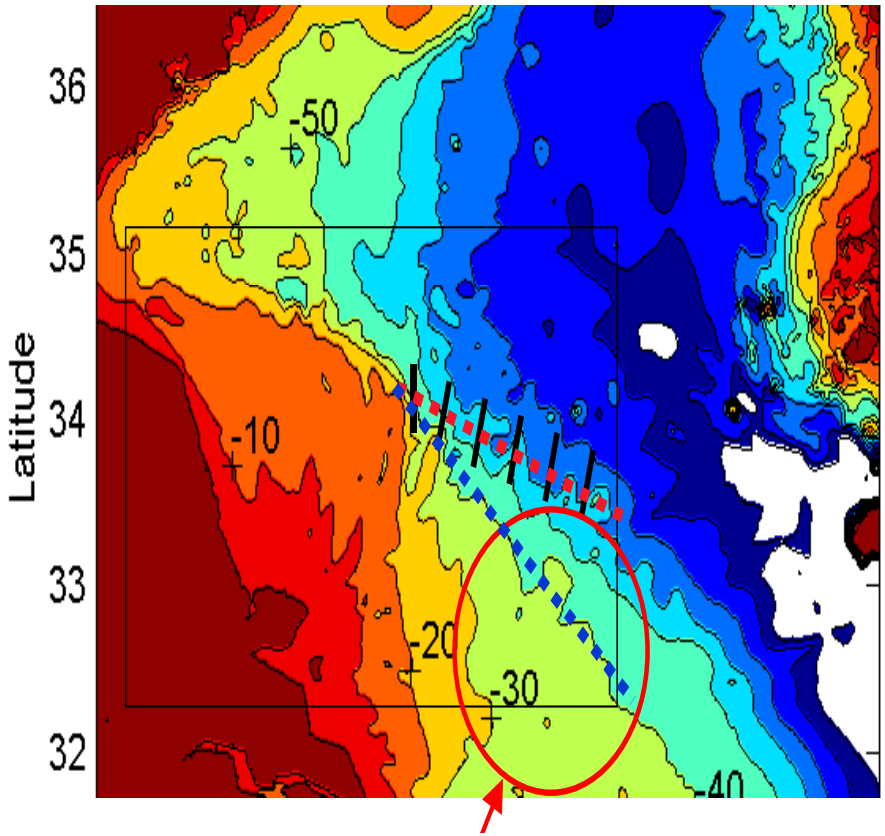


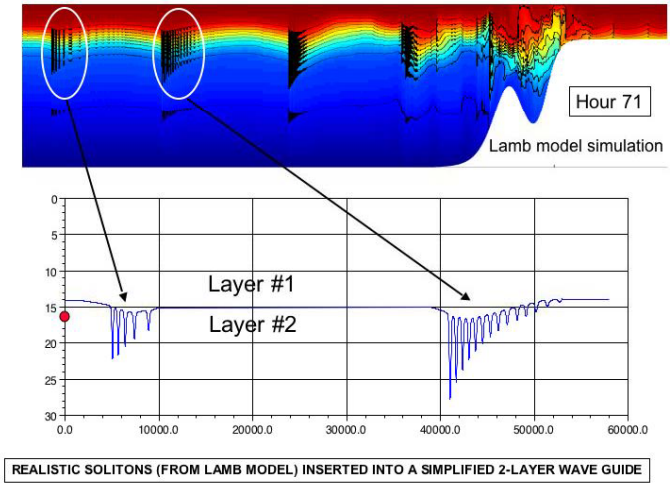
Reference barotropic tide

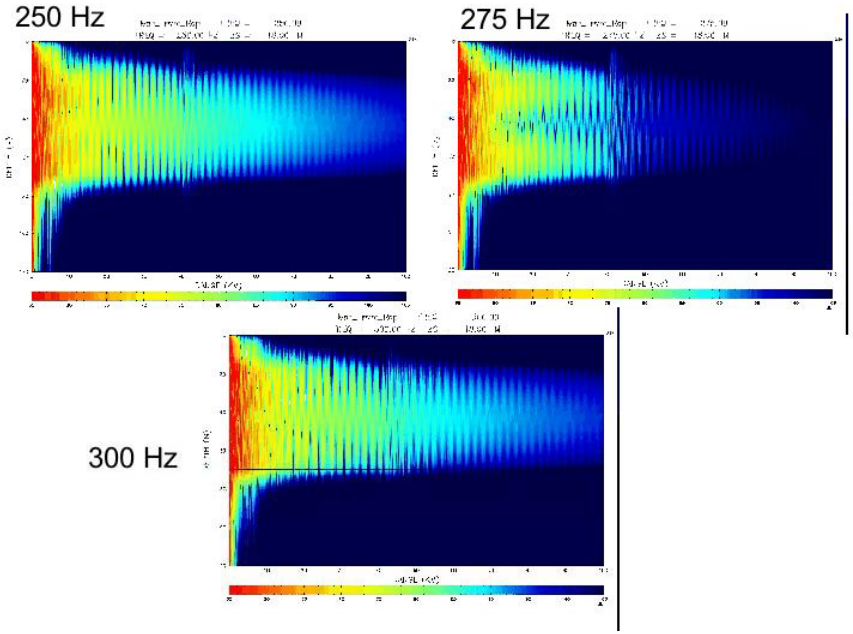


Yellow Sea predictions

Shandong peninsula region





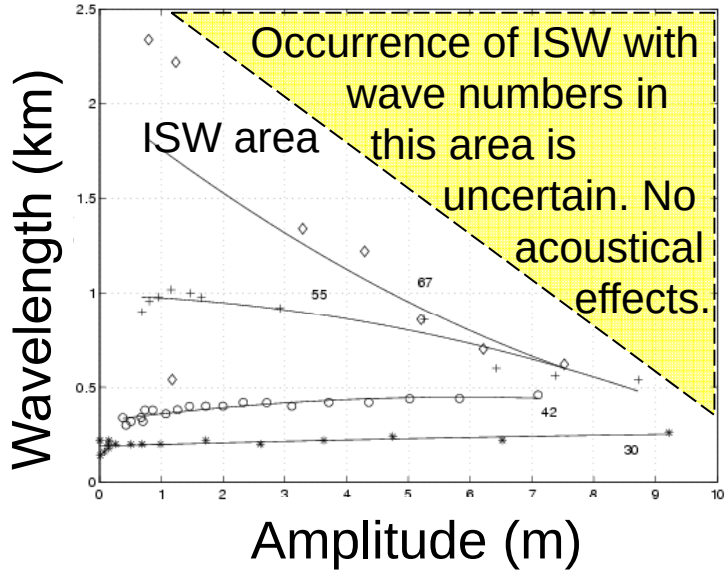


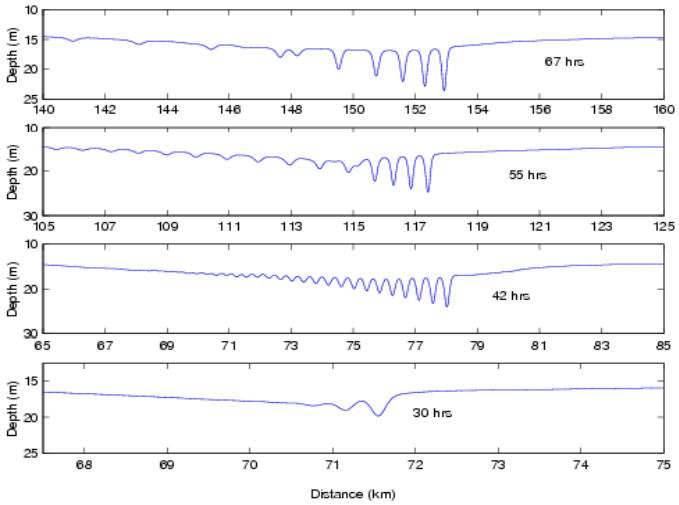
How to Use the Wavenumber Estimate and exploit dispersion Diagrams

$$K_M = k_n - k_{n-1}$$

From the acoustically estimated oceanographic wavenumber:

- Determine likely location in areas's dispersion diagrams.
- If location is in solitary wave signature region, proceed to : estimate solitary wave amplitude and wavelength.
- To reduce uncertainty perform predictions of solitary waves with area's oceanographic parameters





Summary

. The Saint-Venant equations have the same bifurcation values as the classical case, yet no hysteresis.

• Determined apriori hysteresis relationship between 1 st and 2 nd eof coefficients in region of interest where internal solitary waves (ISW) are present.

• Reconstruct ISW trains, corresponding sound speeds and did transmission loss calculations.

• Barotropic tide controls ISW amplitudes.

• Initial stratification, topography and tide determine the half widths and amplitudes of ISW.

• In presence of ISW, at some frequencies acoustical energy can be transferred from higher to lower acoustical wave numbers and attenuate by bottom effects.

• The relationship, $K_M = k_n - k_{n-1}$ ***,can be exploited in conjunction with oceanographic dispersion diagrams of half widths versus amplitude.***

• In presence of ISW a feedback loop exists between acoustics and oceanography.