A hybrid background error covariance model for assimilating glider data

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Motivation:

1. Glider technology provides ***fast and effective data coverage*** of near-coastal domains (an ideal tool for Navy operational needs)

2. Regional models have ***large background errors*** near the coast [if not trained by prior observations] .

3. ***The forecast skill*** crucially depends on the quality of data assimilation = on the quality of the background error covariance model

Improve "THE MATH LENS"
supporting Navy operations in coastal regions:

1. **Develop a background error covariance (BEC) model capable of**
   a) handling large background errors
   b) quick "learning" from intense data flows
   c) adaptively adjusting itself by detection of robust structures in the forecast error fields.
2. **Approach: Hybrid BEC modeling in state space**

Analysis: model space vs data space

$$J = \frac{1}{2}\left[\delta \mathsf{x}^T \mathbf{B}^{-1}\delta \mathsf{x} + (\boldsymbol{H}\delta \mathsf{x} - \delta d)^T \boldsymbol{R}^{-1}(\boldsymbol{H}\delta \mathsf{x} - \delta d)\right] \rightarrow \min_{\delta \mathsf{x}}$$

$$\delta \mathsf{x} = \mathsf{x} - \mathsf{x}_b \qquad \delta d = d - \boldsymbol{H}\mathsf{x}_b$$

Woodbury identity

$$\delta \mathsf{x} = \left[\mathbf{B}^{-1} + \boldsymbol{H}^T \boldsymbol{R}^{-1}\boldsymbol{H}\right]^{-1} \boldsymbol{H}^T \boldsymbol{R}^{-1}\delta d \quad \longleftrightarrow \quad \delta \mathsf{x} = \mathbf{B}\boldsymbol{H}^T\left[\boldsymbol{H}\mathbf{B}\boldsymbol{H}^T + \boldsymbol{R}\right]^{-1}\delta d$$

Model space formulation **Data space formulation**

0. Formulations are equivalent only if the increment δx is restricted to the subspace spanned by the eigenvectors of **B**.
1. The structure of B is poorly known: only a small number of eigenvectors **(if any)** can be captured with confidence.
2. Result of assimilation crucially depends on the model of **B** (or $\mathbf{B}^{-1}$)

Hybrid covariance modelling

$$J(\delta \mathsf{x}) = \frac{1}{2}\left[\delta \mathsf{x}^{\dagger}\mathsf{B}^{-1}\delta \mathsf{x} + (H\delta \mathsf{x} - \delta y)^{\dagger} R^{-1}(H\delta \mathsf{x} - \delta y)\right] \rightarrow \min_{\delta \mathsf{x}}$$

$$\delta y = H\mathsf{x}_b - d$$

$$\mathsf{B}^{-1} = \alpha \mathsf{B}_m^{-1} + \beta \mathsf{B}_0^{-1}$$

$$\mathsf{B}^{-1} = \alpha P \Lambda_m^{-1} P^{\dagger} + \beta P_{\perp} \mathsf{B}_0^{-1} P_{\perp}^{\dagger}$$

$$P_{\perp} = \mathsf{I} - PP^{\dagger}$$

$$\mathsf{B} = \alpha \mathsf{B}_m + \beta \mathsf{B}_0$$

$$\mathsf{B}_0 = \exp(\nabla \nu \nabla t)$$

heuristic (Gaussian)

$$\mathsf{B}_m = P \Lambda_m P^{\dagger}$$

dynamical (derived from model statistics)

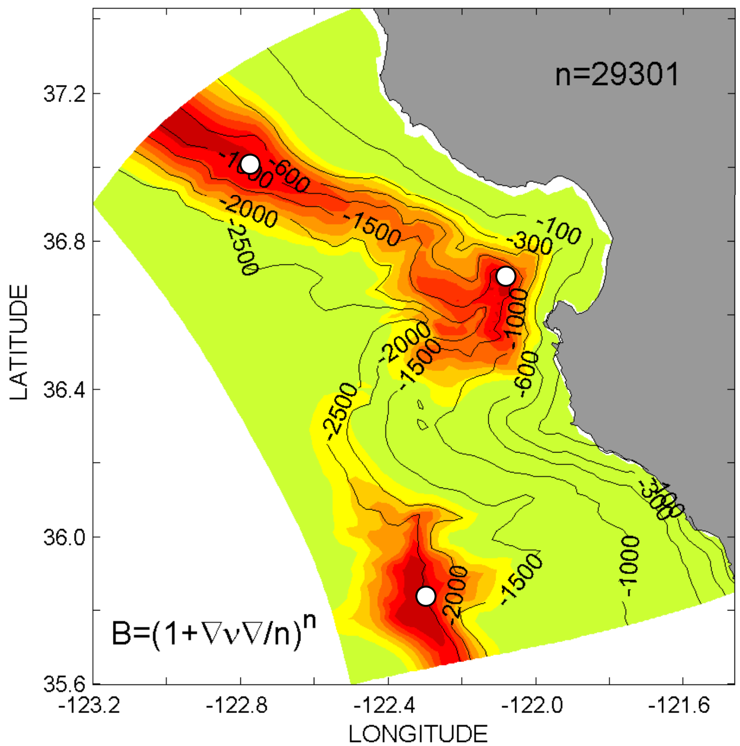
$$\mathsf{B} = \frac{1}{\alpha} P \Lambda_m P^{\dagger} + \frac{1}{\beta}[P_{\perp} \exp(-\mathsf{D}t) P_{\perp}^{\dagger}]^{-1}$$

$$\mathsf{D} = \nabla^{\dagger} \nu \nabla$$

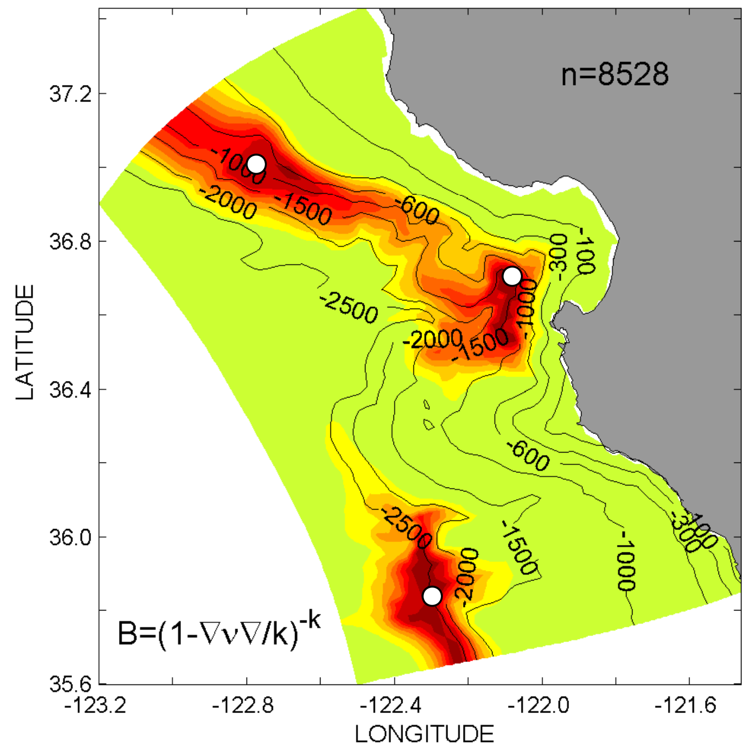
Computation of the heuristic covariance B_0

Explicit vs implicit approximation of the Gaussian operator

$\exp(t\mathbf{D}) \sim (\mathbf{I} + t\mathbf{D}/n)^n$



$\exp(t\mathbf{D}) \sim (\mathbf{I} - t\mathbf{D}/m)^{-m}$



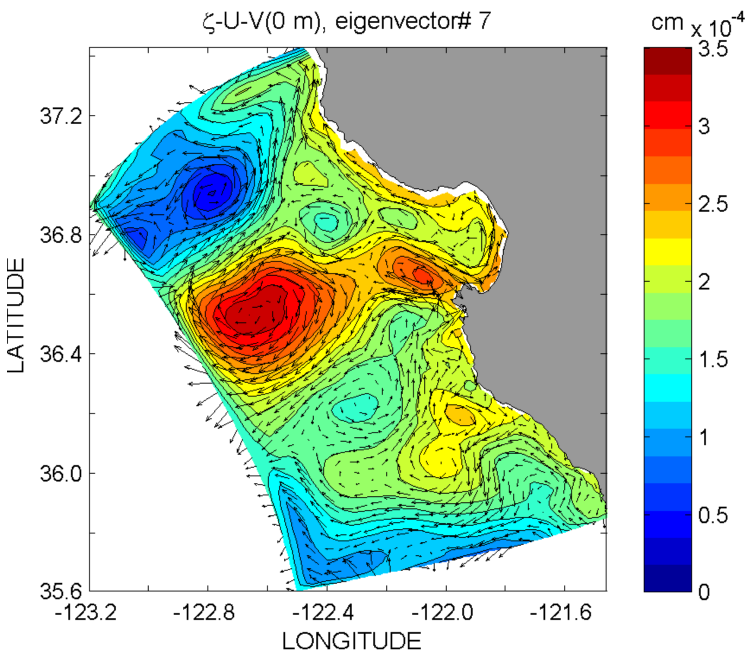
$\max[\rho/\delta x] = 11$ $m=2$ $CPU_{obs} / CPU_{mod} \sim 20m\,[\rho/\delta x]^{1-m} \sim 4$

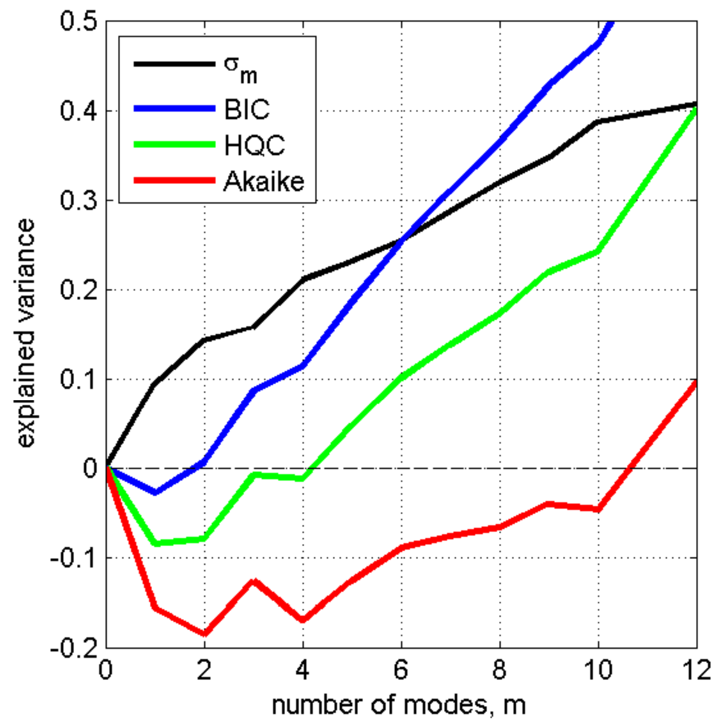
Estimation of the number of modes *m*

$$B^{-1} = \alpha P \Lambda_m^{-1} P^{\dagger} + \beta P_{\perp} B_0^{-1} P_{\perp}^{\dagger}$$

$$\boxed{\begin{aligned} \mathcal{C}_{\mathrm{BIC}}(m) &= m + \frac{N}{\ln N} \ln \sigma_m^2 \\ \mathcal{C}_{\mathrm{Akaike}}(m) &= m + \frac{N}{2} \ln \sigma_m^2 \\ \mathcal{C}_{\mathrm{HQC}}(m) &= m + \frac{N}{2 \ln \ln N} \ln \sigma_m^2 \end{aligned}}$$

$\boldsymbol{\sigma}_m^2$ - *part of the model-data misfit variance described by* **m** *modes (average over N samples)*





Estimation of α (the magnitude of B_m)

$\delta x = P\delta e$

$$J = \frac{1}{2}\left[\delta e^\dagger P^\dagger B^{-1} P\delta e + (HP\delta e - \delta y)^\dagger (HP\delta e - \delta y)\right] \to \min_{\delta e \in \mathcal{R}^m}$$

$$\left[P^\dagger B^{-1} P + Q\right]\delta e = E\delta y$$

$$\left[\alpha\Lambda_m^{-1} + Q\right]\delta e = E\delta y$$

$$E \equiv P^\dagger H^\dagger \qquad P_\perp P = 0$$

$$Q = EE^\dagger \qquad P^\dagger P = I_m$$

$$\langle \delta e \delta e^\dagger \rangle = (\alpha\Lambda_m^{-1} + Q)^{-1} EYE^\dagger (\alpha\Lambda_m^{-1} + Q)^{-1}$$

$$Y = \langle \delta y \delta y^\dagger \rangle$$

$$Y = HBH^\dagger + I_K$$

$$EYE^\dagger = EHBH^\dagger E^\dagger + EE^\dagger = \frac{1}{\alpha} Q\Lambda_m Q^\dagger + \cancel{\frac{1}{\beta} EH[P_\perp B_0^{-1} P_\perp]^{-1} H^\dagger E^\dagger} + Q.$$

$$EYE^\dagger = \frac{1}{\alpha} Q\Lambda_m Q^\dagger + Q$$

$$P^\dagger H^\dagger H[P_\perp B_0^{-1} P_\perp]^{-1} H^\dagger HP \simeq 0$$

$$\langle \delta e \delta e^\dagger \rangle = \left[\alpha\Lambda_m^{-1} + Q\right]^{-1} \left[\frac{1}{\alpha} Q\Lambda_m Q^\dagger + Q\right] \left[\alpha\Lambda_m^{-1} + Q\right]^{-1}$$

$$\langle \delta e \delta e^\dagger \rangle \simeq \frac{1}{\alpha}\Lambda_m$$

$$|Q| \gg \alpha / |\Lambda_m|$$

Estimation of β (the magnitude of B_0)

$$Y = \langle \delta y \delta y^\dagger \rangle = H B H^\dagger + I_K$$

$$\boxed{\beta: \quad \mathrm{Tr}\, Y = \mathrm{Tr}\,(HBH + I_K)}$$

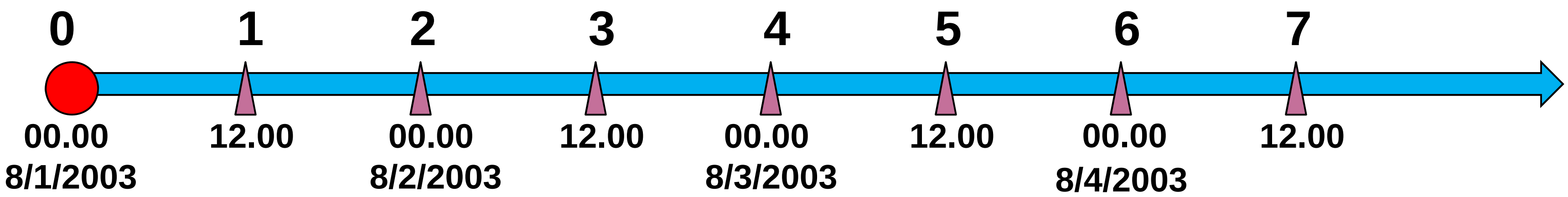
$$\mathrm{Tr}\langle \delta y \delta y^\dagger \rangle = \mathrm{Tr}\left[\frac{1}{\alpha} H P \Lambda_m P^\dagger H^\dagger + \frac{1}{\beta} H [P_\perp \exp(-\mathrm{D}t) P_\perp{}^\dagger]^{-1} H^\dagger\right] + K$$

$$\boxed{\beta = \frac{\mathrm{Tr}\{H[P_\perp \exp(-\mathrm{D}t) P_\perp{}^\dagger]^{-1} H^\dagger\}}{\langle \delta y \delta y^\dagger \rangle - K - \mathrm{Tr}[E^\dagger \Lambda_m E]/\alpha}}$$

ANALYSIS EQUATION

$$\boxed{\left[\alpha P \Lambda_m^{-1} P^\dagger + \beta P_\perp \exp(-\mathrm{D}t) P_\perp^\dagger + H^\dagger H\right] \delta \mathrm{x} = H^\dagger \tilde{d}}$$

Assimilation scheme (1)

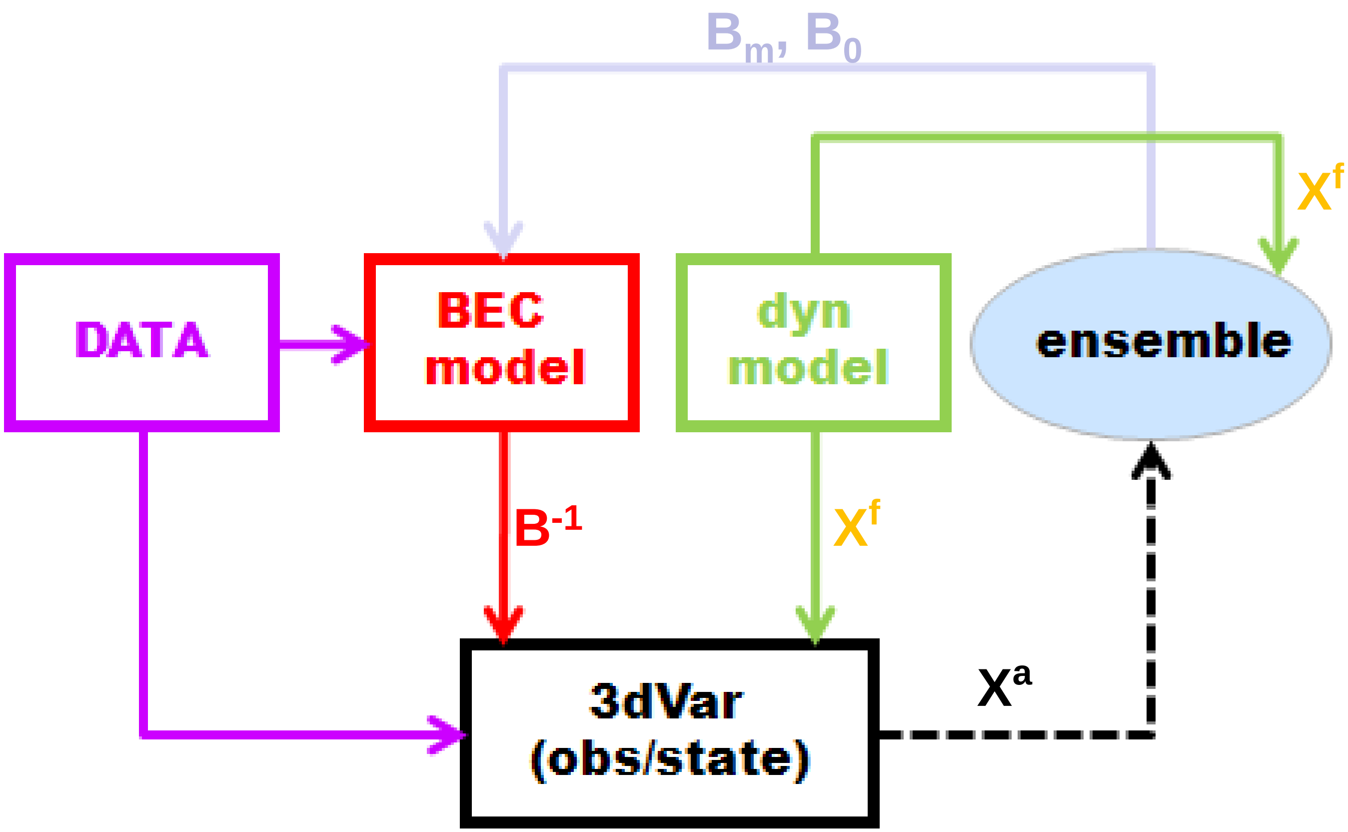


1. $\{X^{a,f}_{1\ldots n-1}\}, B_{n-1} \rightarrow X^{a}_{n}, X^{f}_{n}$ perform analysis + forecast at $t=t_n$

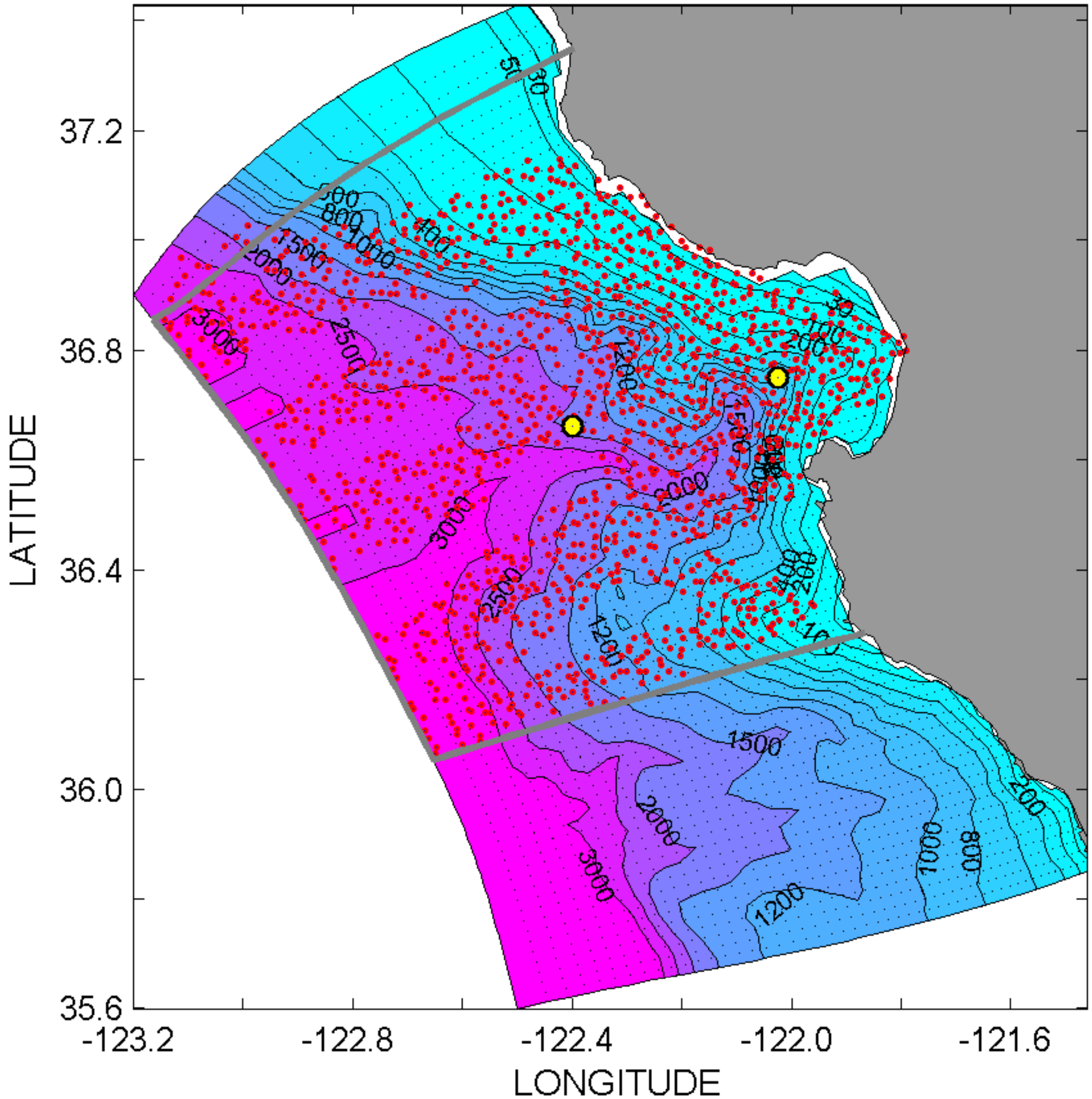
2. $\{X^{f}_{1\ldots n-1}\}, B_{n-1} \rightarrow \{X^{f}_{1\ldots n}\}, B_n$ update the ensemble/covariance

3. $\{X^{a,f}_{1\ldots n}\}, B_n \rightarrow X^{a}_{n+1}, X^{f}_{n+1}$ perform analysis + forecast at $t=t_{n+1}$

Assimilation scheme (2)



Twin data experiments (1)

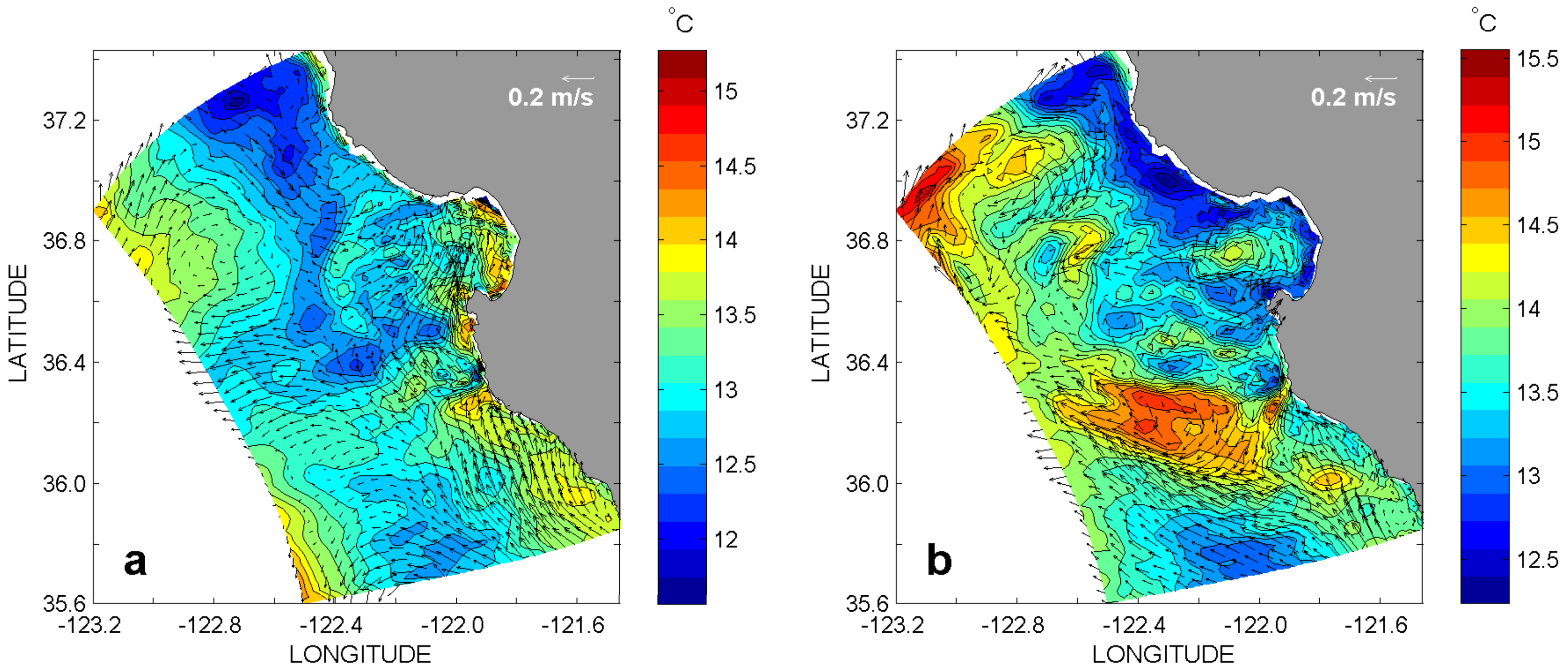


Gliders:
duration: 27 days
K ~1.5 x 10^5

Model:
NCOM 1.5km 40 levels
M = 515,450

Validation:
a) 12hr forecast errors
b) Two moorings with
18 levels U/V
11 levels T/S

Twin data experiments (2)



“True”

First guess

Skill assessment

Metric

$$\mathsf{G} = \mathrm{diag}\{g_T, g_S, g_u, g_v\} \qquad g_\xi(\mathbf{x}) = \overline{\left[\xi(\mathbf{x}) - \overline{\xi(\mathbf{x})}\right]^2}^{1/2}$$

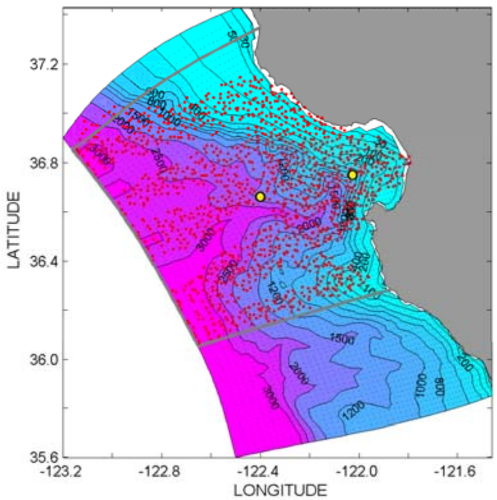
Distances

$$r^s_\xi(\mathsf{x}_1, \mathsf{x}_2) = \langle(\xi_1 - \xi_2)^2 g_\xi^{-2}\rangle_S^{1/2} \qquad r^g_\xi(\mathsf{x}_1, \mathsf{x}_2) = \langle(\xi_1 - \xi_2)^2 R^{-1}\rangle_g^{1/2}$$

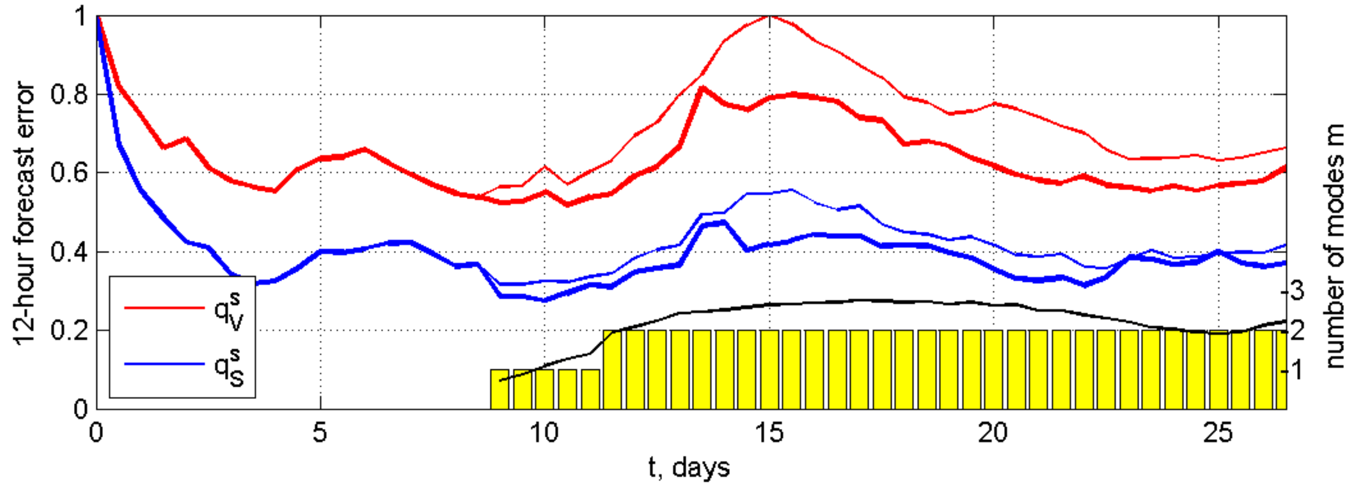
state space — obs space

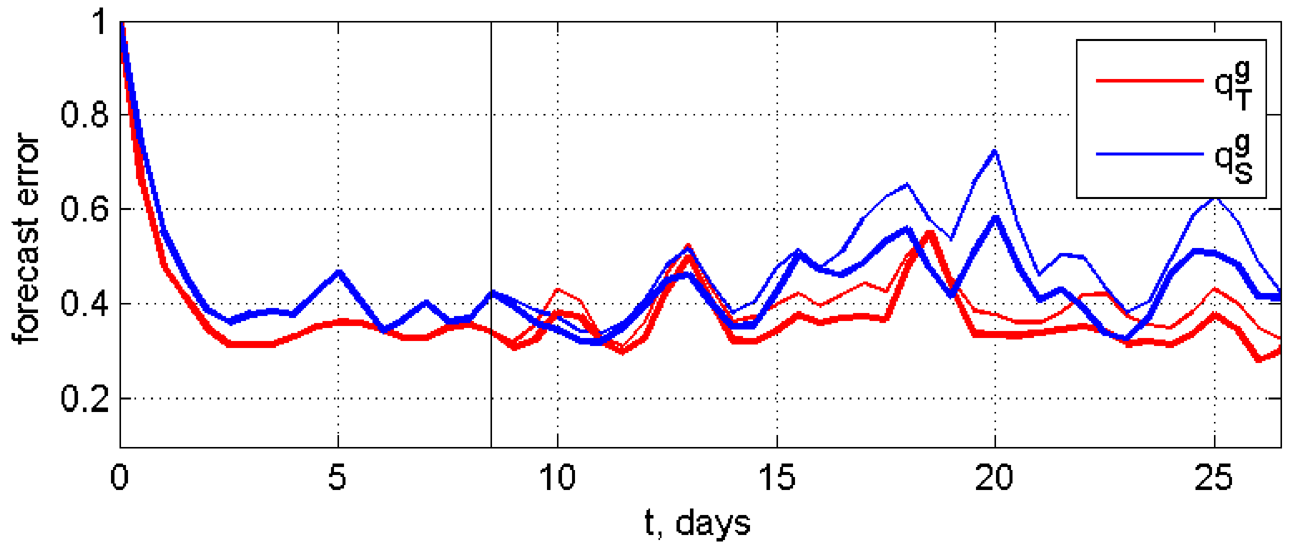
Skill

$$q^{g,s}_\xi(t) = \frac{r^{g,s}_\xi(\mathsf{x}^t, \mathsf{x}^f)|_t}{r^{g,s}_\xi(\mathsf{x}^t, \mathsf{x}^{fg})|_0}$$

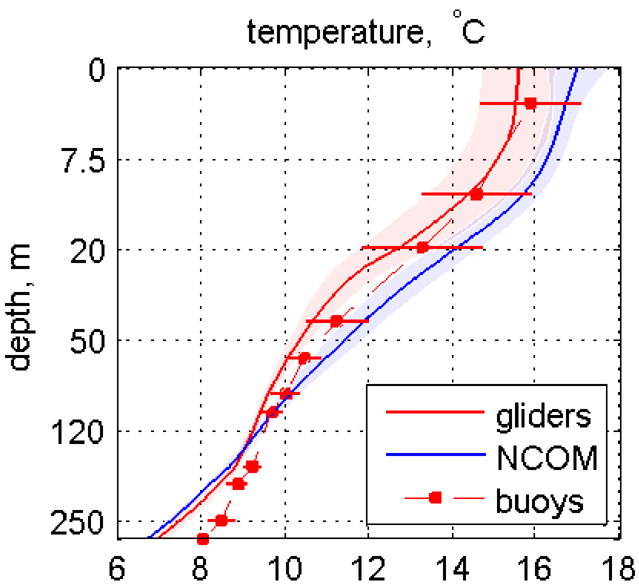


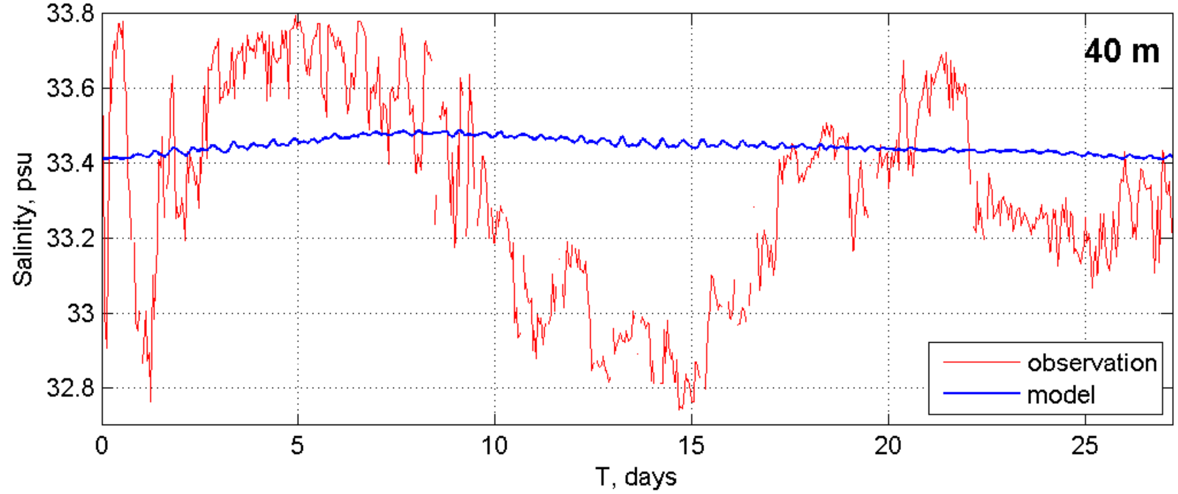
Twin data experiments (3)

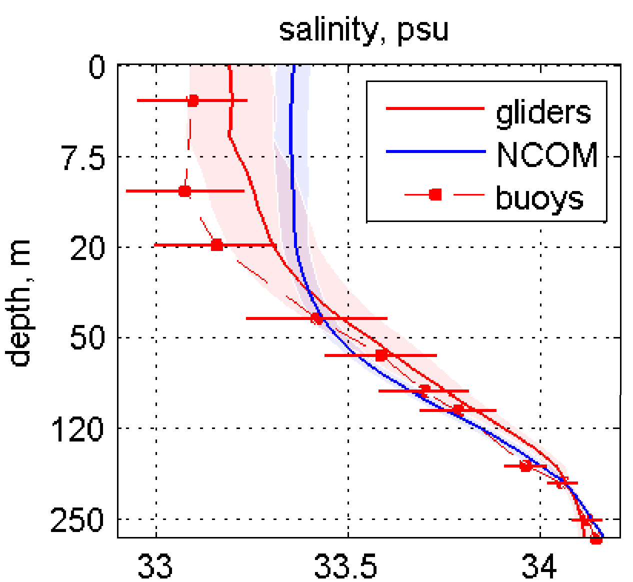




Real data experiments (1)

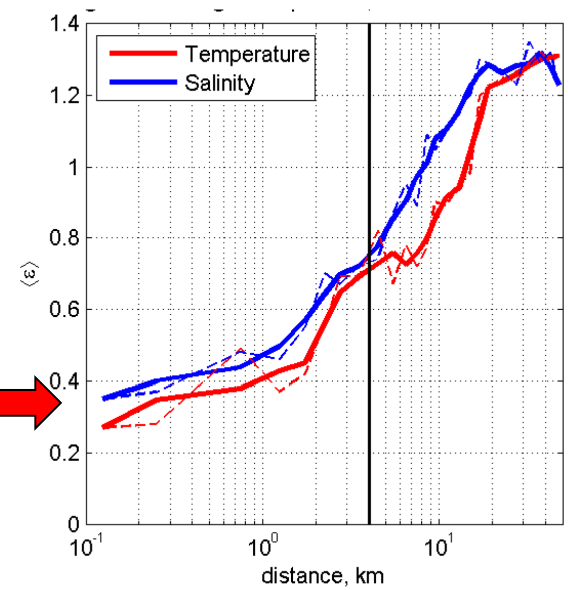




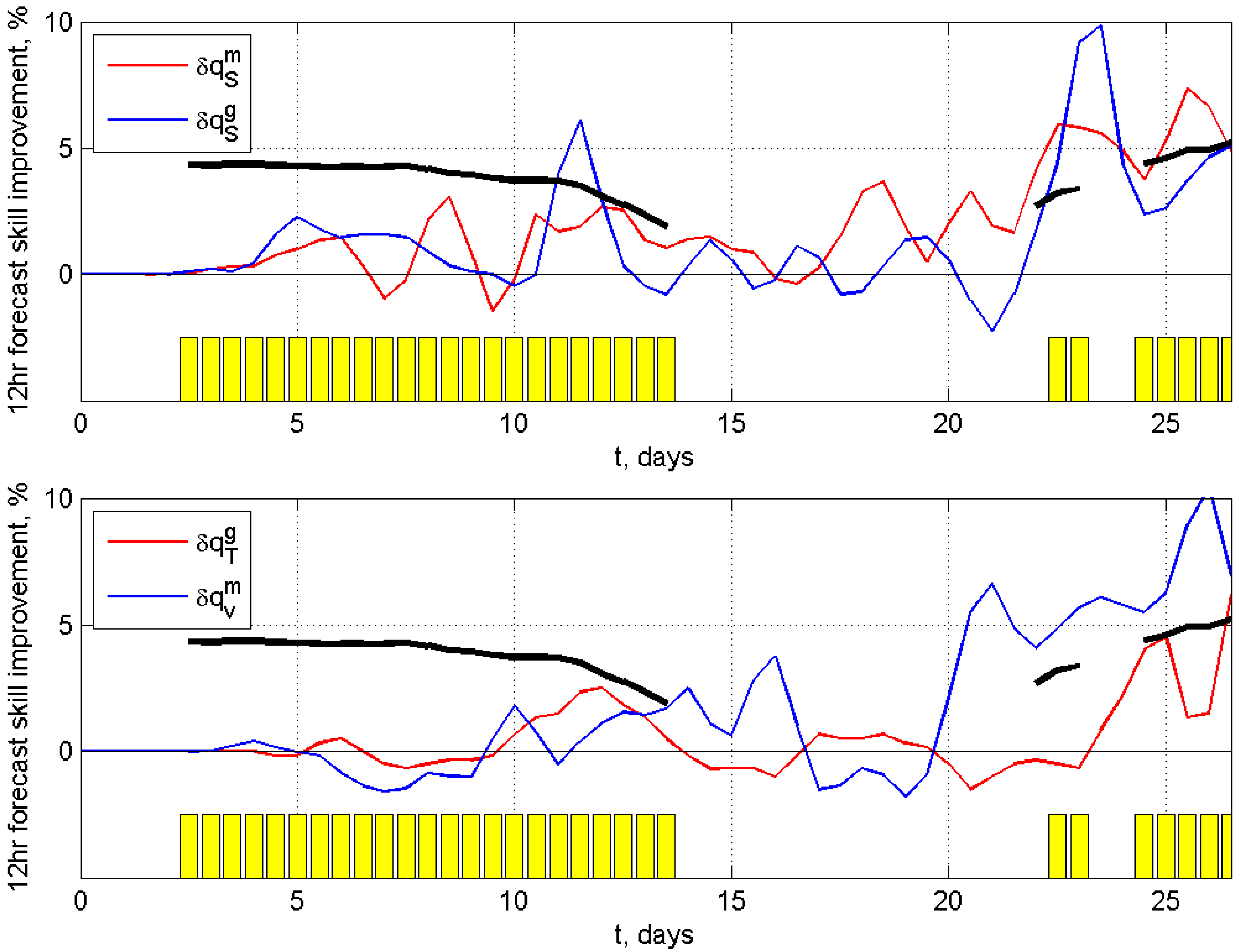


Glider-mooring discrepancies

Obs. Error



Real data experiments (2)



Summary

1. **A hybrid background error covariance model has been proposed and tested with simulated and real glider data in the Monterrey Bay**

2. **The distinctive features are:**
   a) formulation in terms of the inverse error covariances
   b) adaptive determination of the rank($\mathbf{B}_m$) via information criterion
   c) restriction of $\mathbf{B}_0$ to the null space of $\mathbf{B}_m$
   d) adaptive definition of the weighting factors via separate analyses of the innovation vector statistics in the null space of $\mathbf{B}_m$ and its orthogonal supplement

3. **Forecast skill improvement compared to the Gaussian model:**
   a) 15-20% ***in twin-data experiments***
   b) 3-7% ***in real-data experiments***
   [consistent with improvement provided by atmospheric 3dVar hybrids]