Wave forecast with a super-ensemble

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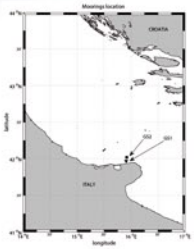
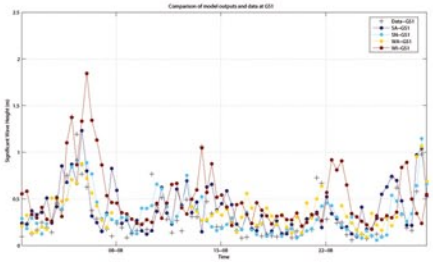
Introduction

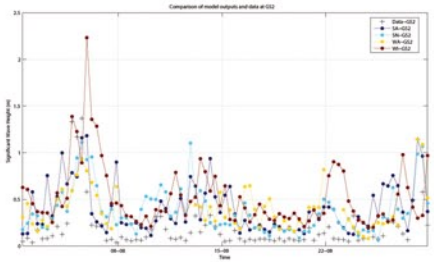
Nowadays, many swell models providing operational forecasts are available. Unfortunately, their results may considerably diverge and choosing one single model among them is a rather difficult and subjective task. A recently developped approach consists in merging different model forecasts into a single multi-model forecast; this technique is called the super-ensemble (SE).

Data and models

During the DART06 campaigns, the SWAN-ARPA, SWAN-NRL, WAM-ATHENS and WAM-ISMAR swell models were run simultaneously besides the significant wave height (SWH), among other parameters, was recorded at different locations of the Adriatic Sea, providing the material necessary to test the SE technique.







Figures

Top left: location of the GS1 and GS2 buoys
Top right: SWH records at GS1 and SWH outputs from the four models during August 2006
Bottom: SWH records at GS2 and SWH outputs from the four models during August 2006

Techniques

To answer the question "How to best combine models in order to improve a 24-h swell forecast?", we consider the following SE methods: the unbiased ensemble mean (UEM), the linear combination of model outputs (LC) and the Kalman filter (KF). They all have in common that they need a learning period (lp), during which observations are used along with model forecasts to derive statistics, that will be used thereafter to make the best hindcast during the lp and forecast during the testing period (tp). They differ in the way information is extracted in the lp and in the way information is utilized in the tp, thus in their complexity.

Let us denote by M the number of data records during the lp and by N the number of models.

UEM

UEM lp: computation of time-averaged values of data and model output timeseries:

$$\overline{y} = \frac{1}{M}\Sigma_{i_{lp}=1}^{M} y_{i_{lp}} \quad \& \quad \overline{x}_j = \frac{1}{M}\Sigma_{i_{lp}=1}^{M} x_{i_{lp},j}$$

UEM tp: sum of the mean observations from the lp and the mean anomalies at hindcast (lp) or at forecast (tp):

$$h_{i_{tp}}^{UEM} = \overline{y} + \frac{1}{N}\Sigma_{j=1}^{N} x'_{i_{tp},j} \quad \text{, where } x'_{i_{tp},j} = x_{i_{tp},j} - \overline{x}_j$$

$$f_{i_{tp}}^{UEM} = \overline{y} + \frac{1}{N}\Sigma_{j=1}^{N} x'_{i_{tp},j} \quad \text{, where } x'_{i_{tp},j} = x_{i_{tp},j} - \overline{x}_j$$

LC

LC lp: model weights are computed by minimizing in the least-square sense the following eq.:

$$\begin{bmatrix} x_{1,1} & \cdots & x_{1,N} & 1 \\ \vdots & x_{i_{lp},j} & \vdots & \vdots \\ x_{M,1} & \cdots & x_{M,N} & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_{N+1} \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix}$$

LC tp: multiplication of model outputs at hindcast (lp) or forecast (tp) by the coefficients derived at the previous step gives the hindcast and forecast estimates:

$$\begin{bmatrix} x_{1,1} & \cdots & x_{1,N} & 1 \\ \vdots & x_{i_{lp},j} & \vdots & \vdots \\ x_{M,1} & \cdots & x_{M,N} & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_{N+1} \end{bmatrix} = \begin{bmatrix} h_1^{LC} \\ \vdots \\ h_M^{LC} \end{bmatrix}$$

$$\begin{bmatrix} x_{1,1} & \cdots & x_{1,N} & 1 \\ \vdots & x_{i_{tp},j} & \vdots & \vdots \\ x_{M,1} & \cdots & x_{M,N} & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_{N+1} \end{bmatrix} = \begin{bmatrix} f_1^{LC} \\ \vdots \\ f_M^{LC} \end{bmatrix}$$

KF

In the case of the SE, the Kalman filter aims to make dynamically evolve the weights.
KF lp: after a normalization of the data and model outputs, initial weights (determined as in the *LC lp* step) are used as the state vector for the KF. The optimal compromise between actual SWH records and simulations is then computed, taking into account the uncertainty of each information source:

$$w_{i_{lp}} = A_{i_{lp}} w_{i_{lp}-1} + \mu_{i_{lp}-1}$$

$$y_{i_{lp}} = H_{i_{lp}} w_{i_{lp}} + \nu_{i_{lp}-1}$$

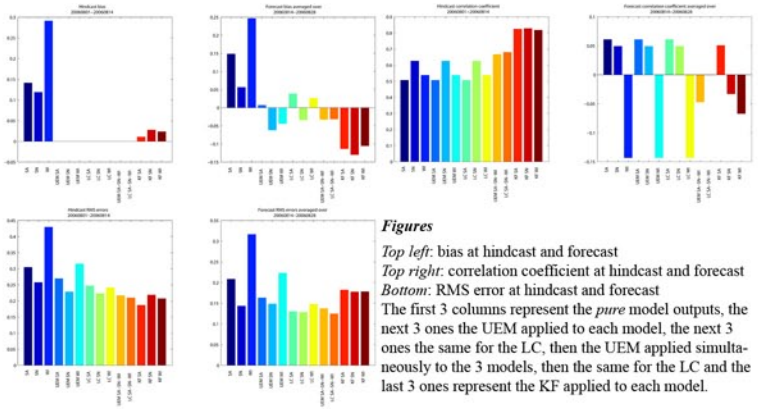
$$\hat{w}_{i_{lp}} = \hat{w}_{i_{tp}}^{-} + K(y_{i_{tp}} - H_{i_{tp}}\hat{w}_{i_{tp}}^{-}), \text{ with } K_{i_{tp}} = P_{i_{tp}} H_{i_{tp}}^T (H_{i_{tp}} P_{i_{tp}} H_{i_{tp}}^T + R_{i_{tp}})^{-1}$$

where A is the state transition model (initialized as an identity matrix),
H is the observation operator (model outputs on the diagonal),
K is the Kalman gain matrix, representing the ratio between model uncertainty (P) and measurement uncertainty (R)
the last term of the two first equations represents respectively the process noise and the observation noise, both are drawn from a normal distribution with a 0-mean and respectively VAR(Q) and VAR(R) for variance.

KF tp: hindcast (lp) and forecast (tp) are computed in the same way as it is done for the linear combination of models (*LC tp* step).

Results

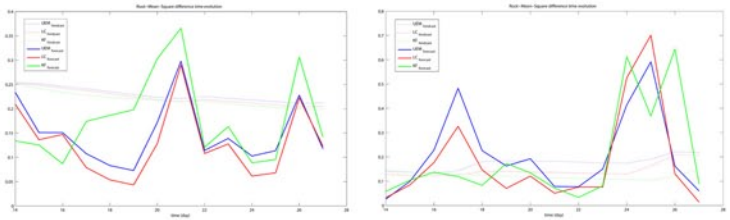
11 SE are tested and their performance are compared with the ones of the *pure* models. The learning period goes from 01/08/2006 to 14/08/2006 for the first 24-h forecast and from 01/08/2006 to 27/08/2006 for the last one. Outputs from SA, SN and WI (interpolated linearly for the spatical dimension and by means of a cubic spline for the time dimenseion) are used to build the SE and data from GS1 and GS2 (about 6 per day) are used for their training, as well as for deriving the statistics that allow an assessment of the techniques quality (bias, correlation coefficient and RMS error). These values are determined on each day of the tp, then averaged all over the latter.

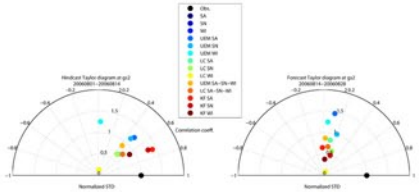


Figures

Top left: bias at hindcast and forecast
Top right: correlation coefficient at hindcast and forecast
Bottom: RMS error at hindcast and forecast
The first 3 columns represent the *pure* model outputs, the next 3 ones the UEM applied to each model, the next 3 ones the same for the LC, then the UEM applied simultaneously to the 3 models, then the same for the LC and the last 3 ones represent the KF applied to each model.

In order to check the robustness of the methods, we virtually modify the dataset by turning back time. Comparative plots of the RMS error time evolution are shown, as well as the hindcast and forecast Taylor diagrams, that summarized the models and SE performance in this synthetic test case.

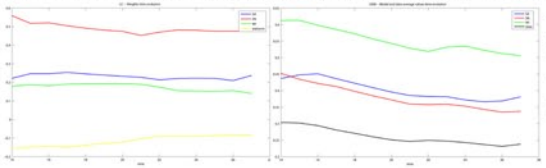




Figures

Top left: RMS error time evolution for the 3 methods applied to the SN outputs in the real case
Top right: RMS error time evolution for the 3 methods applied to the SN outputs in the reversed-time case
Bottom: hindcast and forecast Taylor diagrams for the reversed-time case

Another validity test consists in checking the time-invariance of the coefficients determined at hindcast for the LC and the KF as well as the mean model values for the UEM.



Figures

Left: LC model weights time-evolution
Right: UEM model and data average values time-evolution

Conclusion and perspectives

Swell forecast is a difficult task for which one can be helped by the SE techniques. We have shown that one can obtain an improvement of the prediction by combining different models or just by *unbiasing* a single one. Depending on the situation, one may have about 30% of reduction of the RMS difference between the best single model and the best SE technique forecasts.
Additional filters (*e.g.* EnKF, SIR, *etc.*) could be implemented, a more important *a priori* work could be done on the model outputs in order to shape them for an optimal utilizations of the filters, a few other data sets could be used to enrich the learning period, other model combinations could be performed, *etc.* but in the end only time decides.

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